| SUBJECT: Computational techniques for differential equations |         |           |         |
|--|---------|-----------|---------|
| MASTER: Computational and Applied Mathematics                | ECTS: 6 | COURSE: 1 | TERM: 1 |

|      | WEEKLY PLANNING |   |  |                  |                                       |  |
|------|-----------------|---|--|------------------|---------------------------------------|--|
|      | EK SESSION      | STUDENT WORK DUP  | TUDENT WORK DURING WEEK                          |                  |                                       |  |
| WEEK | SESSION         | SESSION CONTENT   | DESCRIPTION                                      | LECTURE<br>HOURS | STUDENT WORK<br>(Max. 7h per<br>week) |  |
| 1    | 1               | <ul> <li><b>1. FINITE DIFFERENCE METHOD</b> <ol> <li>1.1 Introduction to Finite Difference Approximations             Deriving finite difference approximations; Truncation errors;             1.2 Finite Difference Methods for Steady States and Boundary Value Problems             Finite differences for BVP; Local truncation error; Global error;         </li> </ol></li></ul> | Sections 1 & 2 [LeVeque]                         | 1.66             |                                       |  |
| 1    | 2               | (*) Discussion of select exercises  | (**) Problem solving for selected exercises      | 1.66             | 6.5                                   |  |
| 2    | 3               | 1.2 Finite Difference Methods for Steady States and Boundary Value Problems<br>Stability; Consistency; Convergence; L2 Stability; Boundary conditions; Existence and uniqueness   | Section 2 [LeVeque]                              | 1.66             |                                       |  |
| 2    | 4               | (*) Discussion of select exercises  | (**) Problem solving for selected exercises      | 1.66             | 6.5                                   |  |
| 3    | 5               | 1.3 Finite Difference Methods for Linear Elliptic Equations<br>The 5-point stencil for the Laplacian; Ordering the unknowns and equations; Accuracy and<br>stability; The 9-point Laplacian; Solving the linear system  | Section 3 [LeVeque]                              | 1.66             |                                       |  |
| 3    | 6               | Practice Assignment: Lab 1<br>(*) Discussion of select exercises  | (**) Problem solving for selected exercises      | 1.66             | 6.5                                   |  |
| 4    | 7               | 1.4 Finite Difference Methods for Diffusion Equations and Parabolic Problems<br>Local truncation errors and order of accuracy; Method of lines discretizations; Stability theory;<br>Stiffness of the heat equation; Convergence; Von Neumann analysis; Multidimensional problems;<br>The locally one-dimensional method; Other discretizations   | Section 9 [LeVeque]                              | 1.66             |                                       |  |
| 4    | 8               | (*) Discussion of select exercises  | (**) Problem solving for selected 1.66 exercises |                  | 6.5                                   |  |

| 5  | 9  | 1.5 Finite difference methods for linear advection equation<br>Advection; Method of lines discretization; The Lax-Wendroff method; Upwind methods; Von<br>Neumann analysis; Characteristic tracing and interpolation; The Courant–Friedrichs–Lewy<br>condition   | Section 10 [LeVeque]                        | 1.66 |     |
|----|----|--|---|------|-----|
| 5  | 10 | Practice Assignment: Lab 2<br>(*) Discussion of select exercises   | (**) Problem solving for selected exercises | 1.66 | 6.5 |
| 6  | 11 | 2. THE FINITE ELEMENT METHOD IN 1D<br><u>Piecewise Polynomial Spaces</u> : Interpolation; L2-Projection; Computer Implementation<br><u>Two-point Boundary Value Problem</u> : Variational Formulation; Finite Element Approximation;<br>Derivation of a Linear System of Equations; Basic Algorithm to Compute the Finite Element<br>Solutio; A Priori Error Estimate. | Sections 1 & 2 [Larson&Bengzon]             | 1.66 |     |
| 6  | 12 | (*) Discussion of select exercises   | (**) Problem solving for selected exercises | 1.66 | 6.5 |
| 7  | 13 | <u>Examples</u> : Stationary Heat Equation; Deformation of a Bar; Variable Coefficients<br><u>Computer Implementation</u> : Assembly of the Stiffness Matrix and Load Vector; A Finite Element<br>Solver for a General Two-Point Boundary Value Problem<br><u>Adaptive Finite Element Methods</u> : A Posteriori Error Estimate; Adaptive Mesh Refinement              | Section 2 [Larson&Bengzon]                  | 1.66 |     |
| 7  | 14 | Practice Assignment: Lab 3<br>(*) Discussion of select exercises   | (**) Problem solving for selected exercises | 1.66 | 6.5 |
| 8  | 15 | 3. THE FINITE ELEMENT METHOD IN 2D<br><u>Piecewise Polynomial Approximation in 2D</u> : Meshes; Piecewise Polynomial Spaces; Interpolation;<br>L2-Projection; Quadrature and Numerical Integration; Computer Implementation  | Section 3 [Larson&Bengzon]                  | 1.66 |     |
| 8  | 16 | (*) Discussion of select exercises   | (**) Problem solving for selected exercises | 1.66 | 6.5 |
| 9  | 17 | <u>Green's Formula; The Finite Element Method for Poisson's Equation</u> : Poisson's Equation<br>Variational Formulation; Finite Element Approximation; Derivation of a Linear System of<br>Equations; Basic Algorithm to Compute the Finite Element Solution  | Section 4 [Larson&Bengzon]                  | 1.66 |     |
| 9  | 18 | (*) Discussion of select exercises   | (**) Problem solving for selected exercises | 1.66 | 6.5 |
| 10 | 19 | <u>Basic Analysis of the Finite Element Method</u> : Existence and Uniqueness of the Finite Element Solution; A Priori Error Estimates; Properties of the Stiffness Matrix   | Section 4 [Larson&Bengzon]                  | 1.66 |     |

|          |    | <u>Examples</u> : A Model Problem with Variable Coefficients; Dirichlet Problem; Neumann Problem;<br>Eigenvalue Problem<br><u>Computer Implementation</u> : Assembly of the Stiffness Matrix; Assembling the Boundary Conditions<br><u>Adaptive Finite Element Methods</u> : A Posteriori Error Estimate; Adaptive Mesh Refinement |   |                 |     |
|----------|----|--|---|-----------------|-----|
| 10       | 20 | Practice Assignment: Lab 4<br>(*) Discussion of select exercises   | (**) Problem solving for selected exercises | 1.66            | 6.5 |
| 11       | 21 | 4. SPECTRAL METHODS FOR PERIODIC PROBLEMS     4.1 Differentiation Matrices     4.2 Unbounded Grids: The Semi-Discrete Fourier Transform  | Sections 1 & 2 [Trefethen]                  | 1.66            |     |
| 11       | 22 | (*) Discussion of select exercises   | (**) Problem solving for selected exercises | 1.66            | 6.5 |
| 12       | 23 | <ul><li>4.3 Periodic Grids: The DFT and FFT</li><li>4.4 Smoothness and Spectral Accuracy</li></ul>   | Sections 3 & 4 [Trefethen]                  | 1.66            |     |
| 12       | 24 | (*) Discussion of select exercises   | (**) Problem solving for selected exercises | 1.66            | 6.5 |
| 13       | 25 | 5. SPECTRAL METHODS FOR NON PERIODIC PROBLEMS<br>5.1 Polynomial Interpolation and Clustered Grids<br>5.2 Chebyshev Differentiation Matrices  | Sections 5 & 6 [Trefethen]                  | 1.66            |     |
| 13       | 26 | (*) Discussion of select exercises   | (**) Problem solving for selected exercises | 1.66            | 6.5 |
| 14       | 27 | <ul><li>5.3 Boundary Value Problems</li><li>5.4 Time-dependent problems and stability regions</li></ul>  | Sections 7 & 10 [Trefethen]                 | 1.66            |     |
| 14       | 28 | Practice Assignment: Lab 5<br>(*) Discussion of select exercises   | (**) Problem solving for selected exercises | 1.66            | 6.5 |
| SUBTOTAL |    |  |   | 46.48+91=137.48 |     |

| 15-17 | 27 | Extra sessions, tutorials, etc<br>Preparation for final exam |  | 12.52 |
|-------|----|--|--|-------|
| TOTAL |    |  |  | 150   |

(\*) Discussion from selected exercises from the course collection that correspond to the previous lecture.

(\*\*) Problem solving for selected exercises from the course collection that correspond to the previous lecture.