# **Mathematics**

# September 12 to December 16, 2016 Juan Pablo Rincón-Zapatero

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Lectures: M and W, 9:00–10:30 Rm 15.1.39 Practical Class: F, 12:30–14:00 Rm 15.1.39

# Timetable

## Week 1. Set Theory and the Real Line. Topology in Euclidean spaces

- Session 1: Ordered Sets. Finite, Countable and Uncountable Sets. The Real Field.
- Session 2: Euclidean Spaces. Open, closed and compact sets. Completeness.
- Session 3: (Practical): Resolution of problems about Set Theory, the Real Line and topology in Euclidean spaces.

## Week 2. Numerical Sequences and Series

- Session 1: Convergent Sequences. Subsequences. Cauchy Sequences.
- Session 2: Convergent and Divergent Series. Series of Nonnegative Terms. The Root and Ratio Test. Power Series. Absolute Convergence.
- Session 3: (Practical): Review of Homework 1. Resolution of problems on numerical Sequences and Series.

#### Week 3. Continuity

- Session 1: Limits of Functions. Continuous Functions. Theorems on Continuous Functions.
- Session 2: Monotonic Functions. Convex and Concave Functions.
- Session 3: (Practical): Resolution of problems on continuity of functions.

#### Week 4. Differentiation

- Session 1: The Derivative of a Real Function. Partial and Directional Derivatives. Differentiability.
- Session 2: Inverse and Implicit Function Theorems. Higher Order Derivatives. Taylor's Theorem.
- Session 3: (Practical): Review of Homework 2. Resolution of problems on differentiation of functions.

## Week 5. Integration (I)

- Session 1: Definition and Properties of the Riemann Integral. Fundamental Theorem of Integral Calculus and Barrow's Rule. Iterated integrals.
- Session 2: Improper Integrals. Limitations of the Rieman integral.
- Session 3: (Practical): Resolution of problems on Rieman and improper integrals.

## Week 6. Integration (II)

- Session 1: Introduction to measure theory.
- Session 2: Lebesgue integral.
- Session 3: (Practical): Review of Homework 3. Resolution of problems on measure and Lebesgue integral.

## Week 7. Integration (III)

- Session 1: Theorems of the Monotone Convergence, Fatou and Lebesgue Dominated Convergence. Theorem of Fubini.
- Session 2: Random variables, expectation and conditional expectation.
- Session 3: (Practical): Resolution of problems on convergence and Fubini theorems.

## Week 8. MIDTERM EXAM

#### Week 9. Sequences and Series of Functions

- Session 1: Punctual and Uniform Convergence. Equicontinuity.
- Session 2: Uniform Convergence and Continuity, Differentiation and Integration.
- Session 3: (Practical): Review of Homework 4. Resolution of problems on Sequences and Series of Functions.

# Week 10. Metric Spaces

- Session 1: Distance. Open, Closed and Compact Sets.
- Session 2: Banach and Complete Metric Spaces. Function Spaces.
- Session 3: (Practical): Resolution of problems on metric spaces.

#### Week 11. Fixed Point Theorems of Functions

- Session 1: Theorems of Brower and Schauder-Tychonoff
- Session 2: Contractions mappings. Banach Theorem.
- Session 3: (Practical): Review of Homework 5. Resolution of problems on Fixed Points.

# Week 12. Correspondences

- Session 1: Definition and Properties of Correspondences. Lower and Upper Hemi-Continuous Correspondences.
- Session 2: Alternative characterizations of Lower and Upper Hemi-Continuity.
- Session 3: (Practical): Resolution of problems on correspondences.

## Week 13. Parametric Optimization (I)

- Session 1: Review of optimization techniques.
- Session 2: Maximum Theorem of Berge.
- Session 3: (Practical): Review of Homework 6. Resolution of problems on parametric optimization.

# Week 14. Parametric Optimization (II)

- Session 1: The Concave and the Monotone Maximum Theorem.
- Session 2: Supermodularity. Theorem of Topkins.
- Session 3: (Practical): Resolution of problems on the Maximum Theorem.

## Week 15. FINAL EXAM