Further topics in numerical methods

Academic Year: (2024 / 2025) **Review date: 03-04-2024**

Department assigned to the subject: Mathematics Department

Coordinating teacher: BAYONA REVILLA, VICTOR

Type: Compulsory ECTS Credits : 6.0

Year : 3 Semester : 2

REQUIREMENTS (SUBJECTS THAT ARE ASSUMED TO BE KNOWN)

Linear Algebra (Course 1 - Semester 1); Differential Calculus (Course 1 - Semester 1); Programming (Course 1 - Semester 1); Integral Calculus (Course 1 - Semester 2); Programming Techniques (Course 1 - Semester 2); Numerical Methods (Course 2 - Semester 1); Ordinary Differential Equations (Course 3 - Semester 1).

LEARNING OUTCOMES

CB1. Students have demonstrated possession and understanding of knowledge in an area of study that builds on the foundation of general secondary education, and is usually at a level that, while relying on advanced textbooks, also includes some aspects that involve knowledge from the cutting edge of their field of study.
CB2. Students are able to apply their knowledge to their work or vocation in a professional manner and possess the competences usually demonstrated through the development and defence of arguments and problem solving within their field of study.
CB3. Students have the ability to gather and interpret relevant data (usually within their field of study) in order to make judgements which include reflection on relevant social, scientific or ethical issues.
CB4. Students should be able to communicate information, ideas, problems and solutions to both specialist and non-specialist audiences.
CB5. Students will have developed the learning skills necessary to undertake further study with a high degree of autonomy.
CG1. Students are able to demonstrate knowledge and understanding of concepts in mathematics, statistics and computation and to apply them to solve problems in science and engineering with an ability for analysis and synthesis.
CG2. Students are able to formulate in mathematical language problems that arise in science, engineering, economy and other social sciences.
CG3. Students can solve computationally with the help of the most advanced computing tools mathematical models coming from applications in science, engineering, economy and other social sciences.
CG4. Students are able to show that they can analyze and interpret, with help of computer science, the solutions obtained from problems associated to real world mathematical models, discriminating the most relevant behaviours for each application.
CG5. Students can synthesize conclusions obtained from analysis of mathematical models coming from real world applications and they can communicate in verbal and written form in English language, in an clear and convincing way and with a language that is accessible to the general public.
CG6. Students can search and use bibliographic resources, in physical or digital support, as they are needed to state and solve mathematically and computationally applied problems arising in new or unknown environments or with insufficient information.
CE1. Students have shown that they know and understand the mathematical language and abstract-rigorous reasoning as well as to apply them to state and prove precise results in several areas in mathematics.
CE4. Students have shown that they understand the fundamental results from the theory of ordinary differential equations as well as the theory of partial derivative and stochastic equations.
CE5. Students have shown that they understand basic techniques from numerical calculus, and that they are able to select adequate algorithms for every situation and to program them in a computer.
CE6. Students have shown that they know the fundamental mathematical results supporting the theory and the development of programming languages and intelligent systems.

CE8. Students are able to discretize mathematical models associated to real world problems using interpolation and approximation techniques, in order to solve them numerically by means of direct or iterative methods and to interpret the obtained solutions.
CE9. Students have shown that they can solve mathematical problems derived from new developments in computer science.
CE10. Students have shown that they know and understand the algorithmic procedures to design and build programs that solve mathematical problems paying special attention to performance.

RA1. Students must have acquired advanced cutting-edge knowledge and demonstrated indepth understanding of the theoretical and practical aspects of working methodology in the area of applied mathematics and computing.
RA2. Through sustained and well prepared argument and procedures, students will be able to apply their knowledge, their understanding and the capabilities to resolve problems in complex specialized professional and work areas requiring the use of creative and innovative ideas.
RA3. Students must have the capacity to gather and interpret data and information on which they base their conclusions, including where relevant and necessary, reflections on matters of a social, scientific, and ethical nature in their field of study.
RA4. Students must be able to perform in complex situations that require developing novel solutions in the academic as well as in the professional realm, within their field of study.
RA5. Students must know how to communication with all types of audiences (specialized or not) their knowledge, methodology, ideas, problems and solutions in the area of their field of study in a clear and precise way.
RA6. Students must be capable of identifying their own education and training needs in their field of study and the work or professional environment and organize their own learning with a high degree of autonomy in all types of contexts (structured or not).
RA7. Students must possess the professional maturity necessary to choose and evaluate their work objectives in a reflexive, creative, self-determined and responsible way, for the betterment of society.

DESCRIPTION OF CONTENTS: PROGRAMME

1. Approximation
1.1 Uniform approximation
1.2 Approximation in the 2-norm
1.3 Orthogonal polynomials
1.4 Gaussian quadrature
1.5 Trigonometric interpolation and approximation
1.6 The Fast Fourier Transform (FFT)

2. Computation of eigenvalues and eigenvectors
2.1 The power method
2.2 Jacobi's Method
2.3 Householder transformation; reduction to Hessenberg or tridiagonal forms
2.4 The QR method

3. Ordinary differential equations
3.1 Introduction: existence and uniqueness
3.2 Euler method
3.3 Runge-kutta methods
3.4 Multistep methods
3.5 System of equations
3.6 Stiff problems

LEARNING ACTIVITIES AND METHODOLOGY

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THEORETICAL-PRACTICAL CLASSES. [44 hours with 100% classroom instruction, 1.67 ECTS]
Knowledge and concepts students must acquire. Student receive course notes and will have basic reference texts to facilitate following the classes and carrying out follow up work. Students partake in exercises to resolve practical problems and participate in workshops and evaluation tests, all geared towards acquiring the necessary capabilities.

TUTORING SESSIONS. [4 hours of tutoring with 100% on-site attendance, 0.15 ECTS]
Individualized attendance (individual tutoring) or in-group (group tutoring) for students with a teacher.

STUDENT INDIVIDUAL WORK OR GROUP WORK [98 hours with 0 % on-site, 3.72 ECTS]

WORKSHOPS AND LABORATORY SESSIONS [8 hours with 100% on site, 0.3 ECTS]

FINAL EXAM. [4 hours with 100% on site, 0.15 ECTS]
Global assessment of knowledge, skills and capacities acquired throughout the course.

METHODOLOGIES

THEORY CLASS. Classroom presentations by the teacher with IT and audiovisual support in which the subject's main concepts are developed, while providing material and bibliography to complement student learning.

PRACTICAL CLASS. Resolution of practical cases and problem, posed by the teacher, and carried out individually or in a group.

TUTORING SESSIONS. Individualized attendance (individual tutoring sessions) or in-group (group tutoring sessions) for students with a teacher as tutor.

LABORATORY PRACTICAL SESSIONS. Applied/experimental learning/teaching in workshops and laboratories under the tutor's supervision.

ASSESSMENT SYSTEM

% end-of-term-examination:	40
% of continuous assessment (assigments, laboratory, practicals…):	60

SE1 - FINAL EXAM. [40 %]
Global assessment of knowledge, skills and capacities acquired throughout the course.

SE2 - CONTINUOUS EVALUATION. [60 %]
Assesses papers, projects, class presentations, debates, exercises, internships and workshops throughout the course.

In particular, given the high practical content of this subject, the continuous assessment will consist on:
60% continuous assessment: 3 labs (12% each one) + 1 midterm (24%)
40% final exam

BASIC BIBLIOGRAPHY

- John H. Mathews, Kurtis D Fink Numerical methods using Matlab, Pearson, 2004
- K. Atkinson Elementary Numerical Analysis, Wiley, 2003
- R. L. Burden, J. D. Faires Numerical Analysis, Brooks/Cole, 2010
- Timothy Sauer Numerical Analysis, Pearson, 2012

ADDITIONAL BIBLIOGRAPHY

- A Iserles A First Course in the Numerical Analysis of Differential Equations, Cambridge University Press, 2009

- Endre Süli and David F. Mayers An Introduction to Numerical Analysis, Cambridge, 2003

- Kendall E. Atkinson An Introduction to Numerical Analysis (2nd edition), Wiley, 1989

- Quarteroni, A., Sacco, R., y Saleri, F. Numerical Mathematics, Springer, 2007

- S. D. Conte, Carl de Boor Elementary Numerical Analysis: An Algorithmic Approach , McGraw-Hill , 1980

- Uri M. Ascher, Chen Greif A First Course on Numerical Methods, SIAM, 2011