Numerical Methods

Academic Year: (2024 / 2025) **Review date: 27-06-2024**

Department assigned to the subject: Mathematics Department

Coordinating teacher: TERAN VERGARA, FERNANDO DE

Type: Compulsory ECTS Credits : 6.0

Year : 2 Semester : 1

REQUIREMENTS (SUBJECTS THAT ARE ASSUMED TO BE KNOWN)

Linear Algebra, 1st semester 1st year,
Differential Calculus, 1st semester 1st year,
Integral Calculus, 2nd semester 1st year,
Programming, 1st semester 1st year

SKILLS AND LEARNING OUTCOMES

CB1. Students have demonstrated possession and understanding of knowledge in an area of study that builds on the foundation of general secondary education, and is usually at a level that, while relying on advanced textbooks, also includes some aspects that involve knowledge from the cutting edge of their field of study.
CB2. Students are able to apply their knowledge to their work or vocation in a professional manner and possess the competences usually demonstrated through the development and defence of arguments and problem solving within their field of study.
CB3. Students have the ability to gather and interpret relevant data (usually within their field of study) in order to make judgements which include reflection on relevant social, scientific or ethical issues.
CB4. Students should be able to communicate information, ideas, problems and solutions to both specialist and non-specialist audiences.
CB5. Students will have developed the learning skills necessary to undertake further study with a high degree of autonomy.
CG1. Students are able to demonstrate knowledge and understanding of concepts in mathematics, statistics and computation and to apply them to solve problems in science and engineering with an ability for analysis and synthesis.
CG2. Students are able to formulate in mathematical language problems that arise in science, engineering, economy and other social sciences.
CG3. Students can solve computationally with the help of the most advanced computing tools mathematical models coming from applications in science, engineering, economy and other social sciences.
CG4. Students are able to show that they can analyze and interpret, with help of computer science, the solutions obtained from problems associated to real world mathematical models, discriminating the most relevant behaviours for each application.
CG5. Students can synthesize conclusions obtained from analysis of mathematical models coming from real world applications and they can communicate in verbal and written form in English language, in an clear and convincing way and with a language that is accessible to the general public.
CG6. Students can search and use bibliographic resources, in physical or digital support, as they are needed to state and solve mathematically and computationally applied problems arising in new or unknown environments or with insufficient information.
CE1. Students have shown that they know and understand the mathematical language and abstract-rigorous reasoning as well as to apply them to state and prove precise results in several areas in mathematics.
CE5. Students have shown that they understand basic techniques from numerical calculus, and that they are able to select adequate algorithms for every situation and to program them in a computer.
CE6. Students have shown that they know the fundamental mathematical results supporting the theory and the development of programming languages and intelligent systems.
CE7. Students are able to construct mathematical models of both discrete and continuous processes that appear in real world applications emphasizing the use of deterministic and stochastic difference

and differential equations.
CE8. Students are able to discretize mathematical models associated to real world problems using interpolation and approximation techniques, in order to solve them numerically by means of direct or iterative methods and to interpret the obtained solutions.
CE9. Students have shown that they can solve mathematical problems derived from new developments in computer science.
CE10. Students have shown that they know and understand the algorithmic procedures to design and build programs that solve mathematical problems paying special attention to performance.

RA1. Students must have acquired advanced cutting-edge knowledge and demonstrated indepth understanding of the theoretical and practical aspects of working methodology in the area of applied mathematics and computing.
RA2. Through sustained and well prepared argument and procedures, students will be able to apply their knowledge, their understanding and the capabilities to resolve problems in complex specialized professional and work areas requiring the use of creative and innovative ideas.
RA3. Students must have the capacity to gather and interpret data and information on which they base their conclusions, including where relevant and necessary, reflections on matters of a social, scientific, and ethical nature in their field of study.
RA5. Students must know how to communication with all types of audiences (specialized or not) their knowledge, methodology, ideas, problems and solutions in the area of their field of study in a clear and precise way.

OBJECTIVES

Familiarizing with the basic concepts of numerical analysis: algorithms, stability, accuracy, and efficiency.
Interpolating data with different techniques: Lagrange, Hermite, piecewise, splines.
Calculating numerical approximations, choosing the most adequate algorithm for each application, in each of the following problems: quadrature and derivation, systems of linear and non-linear equations, linear least-squares.
Programming the studied algorithms and use other ready-made algorithms, available in MATLAB or other recognized software packages.
Relating real problems and their mathematical models.

DESCRIPTION OF CONTENTS: PROGRAMME

1. Introduction: errors, algorithms and estimates
Sources of error, roundoff, truncation, propagation. Machine numbers, floating-point arithmetics. Taylor polynomials and error. Estimating and bounding errors. Optimal step. Interval arithmetics.

2. Nonlinear equations and nonlinear systems
Nonlinear equations: Mean-value theorem, number of roots in an interval. Bisection, Secant, Newton-Raphson. Fixed-point methods. Convergence order. Error analysis. Nonlinear systems. Accelerated, Taylor and interpolation methods.

3. Methods for linear systems of equations
Linear systems, stability: condition number. Triangular systems. Gaussian elimination. Pivoting. Computing determinants and matrix inverses. Orthogonalization methods and improved methods. Least-squares problems. Regression. Normal equations and QR method. Overdetermined systems. Fast Fourier Transform. Applications.

4. Polynomial interpolation: Lagrange, Hermite, piecewise, splines
Newton/Lagrange Interpolation, errors. Equispaced (or not) nodes. Runge's phenomenon. Hermite interpolation. Richardson's extrapolation. Splines. Natural cubic splines.

5. Numerical quadrature and differentiation
Numerical differentiation: back/forward, central, general, higher order. Errors. Numerical Integration: Newton-Côtes formulae. Errors. Adaptive integration.

LEARNING ACTIVITIES AND METHODOLOGY

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THEORETICAL-PRACTICAL CLASSES. [44 hours with 100% classroom instruction, 1.67 ECTS]
Knowledge and concepts students must acquire. Student receive course notes and will have basic reference texts to facilitate following the classes and carrying out follow up work. Students partake in exercises to resolve practical problems and participate in workshops and evaluation tests, all geared towards acquiring the necessary capabilities.

TUTORING SESSIONS. [4 hours of tutoring with 100% on-site attendance, 0.15 ECTS]
Individualized attendance (individual tutoring) or in-group (group tutoring) for students with a teacher.

STUDENT INDIVIDUAL WORK OR GROUP WORK [98 hours with 0 % on-site, 3.72 ECTS]

WORKSHOPS AND LABORATORY SESSIONS [8 hours with 100% on site, 0.3 ECTS]

FINAL EXAM. [4 hours with 100% on site, 0.15 ECTS]
Global assessment of knowledge, skills and capacities acquired throughout the course.

METHODOLOGIES

THEORY CLASS. Classroom presentations by the teacher with IT and audiovisual support in which the subject's main concepts are developed, while providing material and bibliography to complement student learning.

PRACTICAL CLASS. Resolution of practical cases and problem, posed by the teacher, and carried out individually or in a group.

TUTORING SESSIONS. Individualized attendance (individual tutoring sessions) or in-group (group tutoring sessions) for students with a teacher as tutor.

LABORATORY PRACTICAL SESSIONS. Applied/experimental learning/teaching in workshops and laboratories under the tutor's supervision.

ASSESSMENT SYSTEM

% end-of-term-examination:	40
% of continuous assessment (assigments, laboratory, practicals…):	60

EVALUATION SYSTEMS

SE1 - FINAL EXAM. [40 %]
Global assessment of knowledge, skills and capacities acquired throughout the course.

SE2 - CONTINUOUS EVALUATION. [60 %]
Assesses papers, projects, class presentations, debates, exercises, internships and workshops throughout the course.

Given that the subject has a highly practical content, a heavier weight (60%) will be assigned to the continuous evaluation. The final grade will be obtained from:
3 practical assignments making up 36% of the grade (12% each);
1 test for 24% of the grade;
1 final exam, summing 40% of the grade.

IMPORTANT: In order to pass the subject, the grade of the final exam obtained by the student must be at least 40% of the maximum grade of the exam (so, for instance, if the maximum grade is 4 points, to get at least 1.6 is mandatory to pass).

BASIC BIBLIOGRAPHY

- [CM] Moler, C. B. Numerical computing with MATLAB, SIAM, 2004

- [KA] Atkinson, K. Elementary Numerical Analysis, John Wiley and Sons, 2004

- [MF] Mathews, J. H., Fink, K. D. Numerical methods using Matlab, 3rd edition, Prentice-Hall, 1998

- [TB] Trefthen, L. N., Bau, D., III Numerical Linear Algebra, SIAM, 1997

ADDITIONAL BIBLIOGRAPHY

- L. N. Trefethen Approximation Theory and Approximation Practice (Extended Ed.), SIAM, 2020

- [HH] Higham, D., Higham, N. MATLAB guide, 2nd edition, SIAM, 2005

- [QSS] Quarteroni, A., Sacco, R., Saleri, F. Numerical mathematics, Springer, 2007