Linear Algebra

Academic Year: (2023 / 2024) **Review date: 14-04-2023**

Department assigned to the subject: Mathematics Department

Coordinating teacher: TORRENTE ORIHUELA, ESTER AURORA

Type: Basic Core ECTS Credits : 6.0

Year : 1 Semester : 1

Branch of knowledge: Engineering and Architecture

REQUIREMENTS (SUBJECTS THAT ARE ASSUMED TO BE KNOWN)

Basic knowledge on vectors and Euclidean space.
Basic knowledge on matrices and determinants.
Basic knowledge on systems of linear equations.
Basic trigonometry.

OBJECTIVES

1. Learning objectives:
- To understand the concept of algebraic structures.
- To know the field of complex numbers and their properties.
- To solve systems of linear equations and to interpret the results.
- To know and understand the notion of vector spaces.
- To understand the notion of bases and coordinates in a vector space, and the change-of-basis and change-of-coordinate problems.
- To understand linear transformation and to represent a linear transformation by a matrix.
- To know and to understand the vector spaces associated to a given matrix.
- To compute the eigenvalues and eigenvectors of a matrix.
- To compute the QR decomposition of a matrix.
- To understand the least squares solution of an inconsistent system.
- To understand the concept of ordinary differential equation and to solve problems related to linear ordinary differential equations with constant coefficients.
- To know dynamic systems and to solve them by using eigenvalues and eigenvectors.
2. Specific skills:
- To raise the abstraction.
- To be able to solve real problems using typical linear algebra tools.
3. General skills:
- To improve the oral and written communication ability using the language and signs of mathematics properly.
- To be able to model a real situation by a linear transformation.
- To improve the ability to interpret a mathematical solution and define its limitations and reliability.
- To be able to use mathematical software.

DESCRIPTION OF CONTENTS: PROGRAMME

1. Complex numbers
   1. Definitions
   2. Forms of complex numbers
   3. Properties and operations

2. Matrices
   1. Operations with matrices
   2. Transpose and inverse of a matrix
   3. Determinants
   4. Sets associated to a matrix

3. Systems of linear equations
   1. Introduction and definitions
   2. Geometric interpretation
   4. Matrix methods to solve linear systems: Gauss and Gauss-Jordan. Application to the inverse of a square matrix

5. Homogeneous systems

4. Vector Spaces
1. Definitions
2. Operations and properties
3. Vector subspaces
4. Spanning sets
5. Linear dependence and independence
6. Bases and dimension. Coordinates. Change of bases. Change of coordinates.

5. Linear Transformations
1. Definition, properties and operations
2. Inverse of a linear transformation
3. Kernel and range of a linear transformation

6. Linear transformations and matrices
1. Representation of linear transformations from IR^m to IR^n with matrices
2. Representation of linear transformations between arbitrary vector spaces

7. Eigenvalues and eigenvectors of a square matrix
1. Definitions
2. Similarity and diagonalisation
3. Spectral theorem

8. Orthogonality
1. Definitions. Inner product. Length of a vector. Angle between two vectors. Orthogonal projection
2. Orthogonal and orthonormal bases
3. Orthogonal matrices and orthogonal linear transformations
4. Orthogonal subspaces and orthogonal complement
5. The Gram-Schmidt process and the QR factorisation

9. Least squares
1. Best approximation in the sense of least squares
2. Computation of the least squares solution
3. Applications to data fit and approximation of functions with polynomials

10. Introduction to Linear Ordinary Differential Equations with constant coefficients
9.1. Introduction to continuous dynamical systems and differential equations
9.2. Linear ordinary differential equations
9.3. Solution to systems of linear ordinary differential equations with constant coefficients
9.4. Introduction to stability of continuous dynamical systems

0. Review Topics
0.1. Introduction to Linear Systems
0.2. Basics vectors and matrix

Complex numbers
1. Definitions
2. Forms of complex numbers
3. Properties and operations

1. Systems of linear equations
1.1. Introduction and definitions
1.2. Geometrical interpretation
1.3. Techniques for solving linear systems
1.4. Matrix methods: Gauss and Gauss-Jordan
1.5. Homogeneous linear system

2. Vector spaces
2.1. Definitions
2.2. Operations and properties
2.3. Vector subspaces
2.4. Linear combinations and Span
2.5. Linear independence
2.6. Bases and dimension of a subspace
2.7. Dot product. Length of a vector. Angle between two vectors
2.8. Orthogonal projection

3. Matrices
   3.1. Operations with matrices
   3.2. Transpose and inverse of a matrix
   3.3. Determinant
   3.4. Matrix subspaces
4. Linear transformations
   4.1 Definitions, properties and operations
   4.2. Inverse of a linear transformation
   4.3. Image and kernel of a linear transformation
5. Bases
   5.1. Coordinates
   5.2. Change of basis
6. Orthogonality
   6.1. Definitions
   6.2. Orthogonal and orthonormal bases
   6.3. Orthogonal matrix and linear transformations
   6.4. Orthogonal projections and orthogonal complements
   6.5. Gram-Schmidt process and QR factorization
7. Least squares
   7.1. Better approximation.
   7.2. Approximation using least squares
   7.3. Methods and applications in data fitting and approximation of functions by polynomials
8. Eigenvalues and eigenvectors
   8.1. Definitions
   8.3. Similarity and Diagonalization
   8.4. Spectral theorem
9. Introduction to Linear Ordinary Differential Equations with Constant Coefficients
   9.1. Introduction to Continuous Dynamical Systems and Differential Equations
   9.2. Linear Ordinary Differential Equations
   9.3. Linear systems of differentiqal equations with constant coefficients
   9.4. Introduction to the Stability of dynamical systems

LEARNING ACTIVITIES AND METHODOLOGY

Lecture sessions (3 credits) (PO: a). During these sessions we will cover the course topics with the aim of using theory to solve problems.

Practicals, working individually and in groups (3 credits) (PO: a) During these sessions we will solve exercises of different levels of difficulty.

ASSESSMENT SYSTEM

We will follow a continuous-assessment system(40%) plus a final exam (60%):

- The continuous-assessment part consists in two written examinations contributing with weight 40% to the final mark.

- The final exam, contributing with weight 60% to the final mark, will be held at the end of the semester. (PO: a.)

% end-of-term-examination:	60
% of continuous assessment (assigments, laboratory, practicals…):	40

BASIC BIBLIOGRAPHY

- B. KOLMAN "Introductory linear algebra: an applied first course", Prentice Hall, 2006
- D. C. LAY "Linear algebra and its applications", Addison-Wesley - 4th ed, 2009
- G. STRANG "Linear Algebra and its applications", Thomson, 2007

ADDITIONAL BIBLIOGRAPHY

- O. BRETSCHER "Linear algebra with applications", Prentice Hall - Segunda edición - 2001.