Stochastic Equations for Finance and Biology

Academic Year: (2023 / 2024) **Review date: 26/04/2023 15:36:53**

Department assigned to the subject: Mathematics Department
Coordinating teacher: BERNAL MARTINEZ, FRANCISCO MANUEL
Type: Compulsory ECTS Credits : 3.0
Year : 1 Semester : 2

REQUIREMENTS (SUBJECTS THAT ARE ASSUMED TO BE KNOWN)

Calculus I
Ordinary Differential Equations
Partial Differential Equations
Probability
Basic programming

OBJECTIVES

CB6, CB7, CB9, CB10
CG1, CG2, CG3, CG5, CG6
CE1, CE3, CE5, CE6, CE7, CE8, CE9, CE11

Understand the basic aspects of stochastic modelling: discrete time models; descriptions of random motion; Brownian motion, models of Einstein and Langevin
Get acquainted with stochastic processes in continuous time, in particular the Wiener process.
Grasp the motivation and subtleties behind the definitions of stochastic integrals, as well as the definition and properties of stochastic differential equations.
Get acquainted with Itô's calculus and its relation with partial differential equations via the Feynman-Kac formula
Understand and know how to program the basic numerical methods for stochastic differential equations and Langevin simulations, as well as the arising numerical errors
Know the most paradigmatic applications of stochastic differential equations in finance and biology

DESCRIPTION OF CONTENTS: PROGRAMME

Part One: introduction to stochastic calculus

1.1 Recap of probability; characteristic functions
1.2 The Law of Large Numbers and the Central Limit Theorem
1.3 Brownian motion; models of Einstein and Langevin
1.4 Wiener process and stochastic integral
1.5 Stochastic differential equations and Itô calculus; paradigmatic SDEs
1.6 Euler-Maruyama method; weak and strong convergence
1.7 Feynman-Kac formula

Part Two: applications in biology

2.1 Stochastic models of population
2.2 Stochastic models of epidemics

Part Three: financial applications

3.1 Financial options; arbitrage; hedging
3.2 Black-Scholes equation; martingales; risk-neutral probability
3.3 Analytical solution; by finite differences; by stochastic simulation

3.4 Implied volatility; Heston's and other advanced models

LEARNING ACTIVITIES AND METHODOLOGY

Class hours will be devoted to the following supervised learning activities:

* Master classes / teacher presentations, in which the main concepts of the course are developed, that students are expected to learn. In order to facilitate this, students will be provided with class notes. Bibliography is also provided to complement the students' learning and enable them to dive further in those topics more interesting to them.

* Practical classes, in which problems are didactically solved, supervised computer practice is carried out in the computer room, or students publicly present their work. These classes help develop specific skills.

Additionally, there will be 2 office hours devoted to tutoring students, consisting in individualised teaching activities of theoretical and practical type, such that they call for closer supervision of a teacher even though they might be carried out autonomously by the student. Such activities may be, among other: scheduled tutorials, correction of student's work, and student mentoring.

The remaining credits are earmarked for student's self-study or group study without teacher supervision. During this time, the student solves proposed exercises and reads supplementary texts suggested by the teacher, as well as other texts from the subject's syllabus. During the time, the student may use the computer room.

ASSESSMENT SYSTEM

% end-of-term-examination/test:	40
% of continuous assessment (assigments, laboratory, practicals…):	60

1) Continuous evaluation, consisting of
- Personal working out and delivery of proposed exercises
- Personal working out and delivery of computer codes used for problems to be solved numerically
- Group solution and exposition of projects (if time and the number of enrolled students allow)

2) Final evaluation: the student's overall knowledge and understanding of the subject will be assessed in a written exam. Its weight in the final mark will be 40%.

BASIC BIBLIOGRAPHY

- Bengt Oksendal Stochastic Differential Equations: An Introduction with Applications (5th Edition), Springer-Verlag, 2014

- Cornelis W. Oosterlee & Lech A. Grzelak Mathematical Modeling and Computation in Finance: With Exercises and Python and MATLAB Computer Codes, World Scientific Publishing Europe Ltd., 2019

- Emmanuel Gobet Monte-Carlo Methods and Stochastic Processes From Linear to Non-Linear, Chapman & Hall, 2020

- Lawrence C. Evans An Introduction to Stochastic Differential Equations, AMS American Mathematical Society, 2013

- Paul Wilmott, Sam Howison & Jeff Dewynne The Mathematics of Financial Derivatives: A Student Introduction, Cambridge University Press, 1995

- Peter E. Kloeden, Eckhard Platen Numerical Solution of Stochastic Differential Equations, Springer-Verlag, 1992

ADDITIONAL BIBLIOGRAPHY

- J. L. García-Palacios Introduction to the theory of stochastic processes and Brownian motion problems Lecture notes for a graduate course,, https://arxiv.org/pdf/cond-mat/0701242.pdf, 2004

- Crispin W. Gardiner Handbook of stochastic methods. Vol. 3., Springer, Berlin, 1985

- Linda J.S. Allen An introduction to stochastic processes with applications to biology, CRC Press, 2010