Modeling and Nonlinear Analysis

Academic Year: (2023 / 2024) **Review date: 17-05-2023**

Department assigned to the subject: Mathematics Department

Coordinating teacher: CUERNO REJADO, RODOLFO

Type: Compulsory ECTS Credits : 6.0

Year : 1 Semester : 1

REQUIREMENTS (SUBJECTS THAT ARE ASSUMED TO BE KNOWN)

Linear Algebra, Calculus (one and several variables), Differential Equations, Probability, Numerical Calculus, and Programming in some language used in science or engineering.

OBJECTIVES

We aim to provide an introduction to important methods and examples of determinstic and stochastic mathematical modeling based on differential and difference equations. Many examples will be discussed, taken from diverse domains of applications, from Natural Science (Physics, Chemistry, Biology) to Engineering, and Social Science. A particular focus will be generic behavior induced by the nonlinear nature of the models studied, such as deterministic chaos, pattern formation. and other. As more specific objectives, we can highlight:

- To be able to formulate a practical model in terms of conservation and constitutive laws, in a consistent way from the point of view of physical dimensions, identifying the main dimensional constants and dimensionless constant ratios characterizing it.
- To become familiar with paradigmatic modeling approaches in Science, Engineering, and Socioeconomic systems through ordinary differential equations, discrete maps, and partial differential equations.
- To become familiar with discrete or continuous-time stochastic models provided by important Markov processes.
- To have a working knowledge of the qualitative theory of dynamical systems.
- To get acquainted with bifurcation phenomena in low-dimensional dynamical systems and partial differential equations.
- To be able to identify and characterize chaotic behavior in discrete and continuous low-dimensional deterministic systems.
- To be acquainted with further nonlinear phenomena in spatially-extended systems, such as reaction-diffusion processes, wave behavior, or pattern formation.

Basic competences: CB6, CB7, CB8, CB9, CB10
General competences: CG1, CG2, CG3, CG4, CG5, CG6, CG7
Specific competences: CE1, CE2, CE3, CE4, CE5, CE6, CE7, CE8, CE9, C11

DESCRIPTION OF CONTENTS: PROGRAMME

1. Introduction to modeling and non-linear analysis.
1.1. Mathematical modeling and nonlinear behavior.
1.2. Dimensional analysis.

2. Dynamical systems.
2.1. Linear systems.
2.2. Phase-plane approach.
2.3. Bifurcations.

3. Deterministic Chaos.
3.1. Phenomenology of chaos.
3.2. One-dimensional maps.
3.3. Characterizations of chaos.

4. Stochastic processes in discrete time.
4.1. Markov chains.
4.2. Branching and renewal processes.

5. Stochastic processes in continuous time.
5.1. Markov processes: Chapman-Kolmogorov equation. Jump and diffusion processes.
5.2. Stationary and homogeneous processes.

6. Spatially-extended systems: Diffusion.
6.1. Transport in continuous media.
6.2. Reaction-diffusion systems.

7. Spatially-extended systems: Traveling waves.
7.1. Fisher-Kolmogorov equation.
7.2. Excitable systems.

8. Spatially-extended systems: Pattern formation.
8.1. Linear stability analysis.
8.2. Nonlinear behavior: amplitude equations.

LEARNING ACTIVITIES AND METHODOLOGY

- Theory sessions: These enable acquiring the specific cognitive competences of the course, via discussion of the theoretical content of the course. To facilitate following these lectures, students will receive related materials (notes, presentations, links), as well as access to the bibliography which allows to complement and/or delve further into various aspects, as needed.

- Practical sessions: These are devoted to solution of examples and exercises, practice in the computer room, or student presentations. These lectures allow to develop specific competences and will alternate with the theoretical ones.

Tutorial learning activities are also foreseen, both of theoretical and practical content, which albeit also suited for autonomous work, require some form of supervision by the Instructor. These activities may include e.g. scheduled or follow-up tutorial sessions, of practice supervision.

Remaining activities are unsupervised autonomous of group work, focused on additional reading, exercises, and practice, with open access to the computer room if required.

ASSESSMENT SYSTEM

The continuous assessment (50%) and the final test (50%) are proposed to have equal weight in the final course score. The former will consist of solving several exercises along the course, some of which may require programming.

% end-of-term-examination:	50
% of continuous assessment (assigments, laboratory, practicals…):	50

BASIC BIBLIOGRAPHY

- M. Cross and H. Greenside Pattern Formation and Dynamics in Non-equilibrium Systems, Cambridge University Press , 2009
- M. H. Holmes Introduction to the Foundations of Applied Mathematics, Springer Science+Business Media, 2019
- M. Pinsky and S. Karlin An Introduction to Stochastic Modeling, Academic Press , 2010
- S. H. Strogatz Nonlinear Dynamics and Chaos, Perseus Books, 2015

ADDITIONAL BIBLIOGRAPHY

- A. Papoulis and S. U. Pillai Probability, Random Variables and Stochastic Processes, McGraw-Hill, 2002
- C. L. Dym Principles of Mathematical Modeling, Elsevier, 2004
- G. Nicolis Introduction to Nonlinear Science, Cambridge University Press, 1995
- I. R. Epstein and J. A. Pojman An Introduction to Nonlinear Chemical Dynamics, Oxford University Press, 1998
- J. D. Logan Applied Mathematics, Wiley Interscience, 2006
- J. D. Murray Mathematical Biology. I, Springer, 2002
- J. D. Murray Mathematical Biology. II, Springer, 2003
- L. Allen An Introduction to Stochastic Processes with Applications to Biology, CRC Press, 2010
- R. C. Desai and R. Kapral Dynamics of Self-organized and Self-assembled structures, Cambridge

University Press, 2009

- S. Heinz Mathematical Modeling, Springer-Verlag, 2011
- S. L. Miller and D. Childers Probability and Random Processes, Elsevier, 2012