Computational Techniques for Differential Equations

Academic Year: (2023 / 2024) **Review date: 10-09-2023**

Department assigned to the subject: Mathematics Department

Coordinating teacher: MOLINA MEYER, MARCELA

Type: Compulsory ECTS Credits : 6.0

Year : 1 Semester : 1

REQUIREMENTS (SUBJECTS THAT ARE ASSUMED TO BE KNOWN)

- Numerical methods at basic level.
- Knowledge of Mathematical Analysis in one and several variables.
- Knowledge of Linear Algebra.
- Knowledge of Ordinary Differential Equations and Partial Differential Equations.
- Basic knowledge of numerical methods for Ordinary Differential Equations.

OBJECTIVES

One of the purposes of this course is to provide the basic techniques for the numerical resolution of EDPs. To do this end, we will analyze and establish the theoretical properties of each method (stability, precision, computational complexity) and we will demonstrate its operation with examples that describe its advantages and disadvantages. The main objective is to develop algorithmic thinking, emphasizing the main computational concepts.

More specifically, the aims of the course with respecty to the students include:
- Understanding of main numerical approximation methods for PDEs: finite difference method; finite element method; spectral methods for periodic and non-periodic problems.
- Ability to analyze the main features of a numerical method: order, stability, convergence.
- Ability to implement numerical methods for the solution of PDEs in one and two dimensions.
- Have criteria to assess and compare different methods depending on the problems to be solved, the computational cost and the presence of errors.
- Ability to program the algorithms studied in the course or use previously programmed algorithms (for example, in Matlab or Python).

CB6, CB7, CB8, CB9, CB10
CG1, CG2, CG3, CG4, CG5, CG6, CG7
CE1, CE2, CE3, CE4, CE5, CE6, CE8, CE9, CE10, CE11, CE12, CE13

DESCRIPTION OF CONTENTS: PROGRAMME

1. The Finite Difference Method
1.1 Introduction to Finite Difference Approximations
1.2 Steady States and Boundary Value Problems
1.3 Linear Elliptic Equations
1.4 Diffusion Equations and Parabolic Problems
1.5 Linear Advection Equations

2. The Finite Element Method in 1D
2.1 Piecewise Polynomial Approximation in 1D
2.2 The Finite Element Method in 1D

3. The Finite Element Method in 2D
3.1 Piecewise Polynomial Approximation in 2D
3.2 The Finite Element Method in 2D

4. Spectral Methods for Periodic Problems
4.1 Differentiation Matrices
4.2 Unbounded Grids: The Semi-Discrete Fourier Transform
4.3 Periodic Grids: The DFT and FFT
4.4 Smoothness and Spectral Accuracy

5. Spectral Methods for Non-Periodic Problems
5.1 Polynomial Interpolation and Clustered Grids
5.2 Chebyshev Differentiation Matrices
5.3 Boundary Value Problems
5.4 Time-dependent problems and stability regions

LEARNING ACTIVITIES AND METHODOLOGY

Theory sessions: These enable acquiring the specific cognitive competences of the course, via discussion of the theoretical content of the course. To facilitate following these lectures, students will receive related materials (notes, presentations, links), as well as access to the bibliography which allows to complement and/or delve further into various aspects, as needed.

Practical sessions: These are devoted to solution of examples and exercises, practice in the computer room, or student presentations. These lectures allow to develop specific competences and will alternate with the theoretical ones.

Tutorial learning activities are also foreseen, both of theoretical and practical content, which albeit also suited for autonomous work, require some form of supervision by the Instructor. These activities may include e.g. scheduled or follow-up tutorial sessions, of practice supervision.

Remaining activities are unsupervised autonomous of group work, focused on additional reading, exercises, and practice, with open access to the computer room if required.

ASSESSMENT SYSTEM

Throughout the semester, students will have to submit a series of exercises solved individually or in groups, which may require programming. The average mark of these exercises will constitute 60% of the final mark. The remaining 40% will correspond to a final exam. In the extraordinary call, the mark of the continuous assessment will be averaged with the extraordinary exam.

% end-of-term-examination:	40
% of continuous assessment (assigments, laboratory, practicals…):	60

BASIC BIBLIOGRAPHY

- Lloyd N. Trefethen Spectral Methods in Matlab, SIAM, 2000
- Mats G. Larson and Fredrik Bengzon The Finite Element Method: Theory, Implementation, and Applications, Springer, 2013
- Randall J. LeVeque Finite Difference Methods for Ordinary and Partial Differential Equations, SIAM, 2007

ADDITIONAL BIBLIOGRAPHY

- C. Canuto, A. Quarteroni, M. Y. Hussaini and T. A. Zang Spectral Methods: Fundamentals in Single Domains, Springer, 2006
- David Gottlieb and Steven A. Orszag Numerical Analysis of Spectral Methods: Theory and Applications, SIAM, 1977
- G. D. Smith Numerical Solution of Partial Differential Equations: Finite Difference Methods, Clarendon Press, 1985
- J. W. Thomas Numerical Partial Differential Equations: Finite Difference Methods, Springer, 1995
- Jan S. Hesthaven, Sigal Gottlieb and David Gottlieb Spectral Methods for Time-Dependent Problems, SIAM, 2007
- John C. Strikwerda Finite Difference Schemes and Partial Differential Equations, SIAM, 2004
- Lloyd N. Trefethen Finite Difference and Spectral Methods for Ordinary and Partial Differential Equations , Cornell University, 1996
- Mark S. Gockenbach Partial Differential Equations: Analytical and Numerical Methods, SIAM, 2011
- Susanne C. Brenner and L. Ridgway Scott The Mathematical Theory of Finite Element Methods, Springer, 2008