# uc3m Universidad Carlos III de Madrid

# Integration and Measure

Academic Year: (2023 / 2024) Review date: 09-02-2024

Department assigned to the subject: Mathematics Department Coordinating teacher: ROMERA COLMENAREJO, ELENA

Type: Compulsory ECTS Credits: 6.0

Year: 2 Semester: 1

### REQUIREMENTS (SUBJECTS THAT ARE ASSUMED TO BE KNOWN)

Linear Algebra (Course : 1 Semester : 1), Differential Calculus (Course : 1 Semester : 1), Integral Calculus (Course : 1 Semester : 2), Vector Calculus (Course : 1 Semester : 2).

#### SKILLS AND LEARNING OUTCOMES

CB1. Students have demonstrated possession and understanding of knowledge in an area of study that builds on the foundation of general secondary education, and is usually at a level that, while relying on advanced textbooks, also includes some aspects that involve knowledge from the cutting edge of their field of study.

CB2. Students are able to apply their knowledge to their work or vocation in a professional manner and possess the competences usually demonstrated through the development and defence of arguments and problem solving within their field of study.

CB3. Students have the ability to gather and interpret relevant data (usually within their field of study) in order to make judgements which include reflection on relevant social, scientific or ethical issues.

CB4. Students should be able to communicate information, ideas, problems and solutions to both specialist and non-specialist audiences.

CB5. Students will have developed the learning skills necessary to undertake further study with a high degree of autonomy.

CG1. Students are able to demonstrate knowledge and understanding of concepts in mathematics, statistics and computation and to apply them to solve problems in science and engineering with an ability for analysis and synthesis. CG2. Students are able to formulate in mathematical language problems that arise in science, engineering, economy and other social sciences.

CG5. Students can synthesize conclusions obtained from analysis of mathematical models coming from real world applications and they can communicate in verbal and written form in English language, in an clear and convincing way and with a language that is accessible to the general public.

CG6. Students can search and use bibliographic resources, in physical or digital support, as they are needed to state and solve mathematically and computationally applied problems arising in new or unknown environments or with insufficient information.

CE1. Students have shown that they know and understand the mathematical language and abstract-rigorous reasoning as well as to apply them to state and prove precise results in several areas in mathematics.

CE2. Students have shown that they understand the fundamental results from real, complex and functional mathematical analysis.

RA1. Students must have acquired advanced cutting-edge knowledge and demonstrated indepth understanding of the theoretical and practical aspects of working methodology in the area of applied mathematics and computing.

RA4. Students must be able to perform in complex situations that require developing novel solutions in the academic as well as in the professional realm, within their field of study.

RA5. Students must know how to communication with all types of audiences (specialized or not) their knowledge, methodology, ideas, problems and solutions in the area of their field of study in a clear and precise way.

# **OBJECTIVES**

To introduce the student in the study of modern integration methods, in particular the Lebesgue integral.

To know the convergence theorems on integration and the functional L^p spaces.

To apply these results to the differentiation of parametric integrals and in particular to the Fourier and Laplace transforms.

#### **DESCRIPTION OF CONTENTS: PROGRAMME**

- Integrals on curves and surfaces
- 2. Green's, Stokes' and Gauss' theorems
- 3. Set measure
- 4. The Lebesgue Integral
- 5. Monotone and dominated convergence
- 6. Lp spaces
- 7. Parametric integrals
- 8. Integral transforms: Laplace and Fourier

#### LEARNING ACTIVITIES AND METHODOLOGY

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THEORETICAL-PRACTICAL CLASSES. [44 hours with 100% classroom instruction, 1.76 ECTS] Knowledge and concepts students must acquire. Student receive course notes and will have basic reference texts to facilitate following the classes and carrying out follow up work. Students partake in exercises to resolve practical problems and participate in workshops and evaluation tests, all geared towards acquiring the necessary capabilities.

TUTORING SESSIONS. [4 hours of tutoring with 100% on-site attendance, 0.16 ECTS] Individualized attendance (individual tutoring) or in-group (group tutoring) for students with a teacher.

STUDENT INDIVIDUAL WORK OR GROUP WORK [98 hours with 0 % on-site, 3.92 ECTS]

FINAL EXAM. [4 hours with 100% on site, 0.16 ECTS] Global assessment of knowledge, skills and capacities acquired throughout the course.

#### **METHODOLOGIES**

THEORY CLASS. Classroom presentations by the teacher with IT and audiovisual support in which the subject's main concepts are developed, while providing material and bibliography to complement student learning.

PRACTICAL CLASS. Resolution of practical cases and problems, posed by the teacher, and carried out individually or in a group.

TUTORING SESSIONS. Individualized attendance (individual tutoring sessions) or in-group (group tutoring sessions) for students with a teacher as tutor.

## ASSESSMENT SYSTEM

# **EVALUATION SYSTEMS**

#### SE1 - FINAL EXAM. [60 %]

Global assessment of knowledge, skills and capacities acquired throughout the course.

# SE2 - CONTINUOUS EVALUATION. [40 %]

Assesses papers, projects, class presentations, debates, exercises, internships and workshops throughout the course.

#### % end-of-term-examination: 50

% of continuous assessment (assignments, laboratory, practicals...): 50

#### **BASIC BIBLIOGRAPHY**

Folland, G.B. Fourier Analysis and its Applications, Wadsforth & Brooks/Cole, 1992

- Marsden, J.E., Tromba, A,J. Vector Calculus, W.H. Freeman and Company, 2003
- Rudin, W. Real and complex Analysis, Mc Graw-Hill (International Student Edition), 1970

### ADDITIONAL BIBLIOGRAPHY

- Apostol, T.M. Mathematical Analysis, Addison-Wesley, 1974
- Bauer, H. Measure and Integration Theory, Walter De Gruyter, 2001
- Beerends, R.J., ter Morsche, H.G., vanden Berg, J.C., van de Vrie, E.M. Fourier and Laplace Transforms, Cambridge University Press, 2003
- Bogachev, V.I. Measure Theory, Volume I, Springer, 2007
- Gamkrelidze (Ed.) Analysis I (Encyclopaedia of Mathematical Sciences, Volume 13), Springer-Vergal, 1989
- Guzmán, M., Rubio, B. Integración, Teoría y Técnicas, Alhambra, 1979
- Leadbette, R., Cambanis, S., Pipiras, V. A Basic Course in Measure and Probability, Cambridge University Press,
- Pao, K., Soon, F., Marsden, J.E., Tromba, A.J. Vector Calculus (Solved Problems), W.H.Freeman & Co Ltd, 1989
- Pestana, D., Rodriguez, J.M., Marcellán, F. Curso Práctico de Variable Compleja y Teoría de Transformadas, Pearson, 2014