# uc3m Universidad Carlos III de Madrid

#### **Advanced Mathematics**

Academic Year: (2023 / 2024) Review date: 25-04-2023

Department assigned to the subject: Mathematics Department

Coordinating teacher: MOLINA MEYER, MARCELA

Type: Basic Core ECTS Credits: 6.0

Year: 2 Semester: 1

Branch of knowledge: Engineering and Architecture

#### REQUIREMENTS (SUBJECTS THAT ARE ASSUMED TO BE KNOWN)

Calculus I, Calculus II, Linear Algebra

#### **OBJECTIVES**

The objective of this course in differential equations is to provide the students with a formal approach to several problems related with physics and engineering. The topics discussed along the subject are usually applied in several disciplines related with the engineering, for example the method of characteristics together with the wave equation is used to model the shock waves experienced by aircrafts after trespassing the sonic barrier or to model the pressure wave generated within a pipe when a valve is suddenly closed in a flowing liquid. The heat equation is of interest to model and design all kind of aerospace devices such us the thermal engines mappings or the dimension of the aircraft heat needs in cabin. The Laplace equation is of help to model all kind of stationary problems, for instance to determine the velocity profile generated by an aerodynamic profile in an homogeneous liquid like air. Additionally the differential equations, both ordinary and partial, are used as well in elasticity, vibrations, solid mechanics, electrical circuits and electronics to model different aspects from the deformation of a solid to the transitory phenomena in a circuit.

# SPECIFIC LEARNING OBJECTIVES (PO a):

- To understand the Linear operators and the principle of superposition for solving differential equations.
- To solve elementary differential equations by separation of variables and other methods.
- To understand the different application scopes of the differential equation to engineering and physics.
- To distinguish between elliptic, hyperbolic and parabolic partial differential equations and which initial or boundary conditions are appropriate for them.
- To understand how to apply separation of variables and the Fourier method to solve initial-boundary value problems for the equations of Mathematical Physics.
- To understand the separation of variables technique, the role of the resulting eigenvalue problems and the principle of superposition to solve initial-boundary value problems for the equations of Mathematical Physics.
- To understand when and how to use the method of characteristics to solve different cases of partial differential equations.

## GENERAL ABILITIES (PO a, g, k):

- To understand the necessity of abstract thinking and formal mathematical proofs.
- To acquire communicative skills in mathematics.
- To acquire the ability to model real-world situations mathematically, with the aim of solving practical problems.
- To improve problem-solving skills.

(PO: a)

## **DESCRIPTION OF CONTENTS: PROGRAMME**

- 1. Introduction
- 1.1 Basic models; direction fields
- 1.2 Classification of differential equations
- 2. First Order Differential Equations
- 2.1 Linear equations; integrating factors
- 2.2 Separable equations
- 2.3 Exact equations

- 3. Second Order Linear Equations
- 3.1 Definitions and examples
- 3.2 Linear homogeneous equations
- 3.3 Homogeneous equations with constant coefficients
- 3.4 Inhomogeneous equations: undetermined coefficients
- 3.5 Variation of constants
- 4. Systems of First Order Linear Equations
- 4.1 Basic theory; higher-order equations
- 4.2 Explicit solutions of non-homogeneous linear systems
- 4.3 Planar linear systems
- 5. Nonlinear Systems and Stability
- 5.1 Planar nonlinear systems
- 5.2 Stability
- 5.3 Periodic solutions
- 5.4 Higher-dimensional systems
- 6. Partial Differential Equations: Introduction
- 6.1 Examples and physical derivation
- 6.1 Types of equations and data; well- vs ill-posed problems
- 7. Separation of Variables
- 7.1 Problem resolution by separation of variables
- 7.2 Fourier trigonometric series: basic properties
- 8. Boundary-value Problems
- 8.1 Sturm-Liouville problems
- 8.2 Self-adjoint operators and spectrum
- 8.3 Rayleigh ¿s quotient
- 8.4 Generalized Fourier series

### LEARNING ACTIVITIES AND METHODOLOGY

Theory (3.0 credits. PO a, g).

Problem sessions working individually and in groups (3.0 credits. PO a, g).

Collective office hours might be offered if the professor deems them to be appropriate.

## ASSESSMENT SYSTEM

We follow a continuous-assessment system plus a final exam:

- The continuous-assessment part consists of 2 written mid term exams with a contribution of 40% of the final score.
- The final exam (with contributing weight of 60% to the final score) will be held at the end of the semester.

% end-of-term-examination: 60 % of continuous assessment (assignments, laboratory, practicals...): 40

# BASIC BIBLIOGRAPHY

- J. C. Robinson An Introduction to Ordinary Differential Equations, Cambridge University Press, 2004
- J.R.Brannan, W.E.Boyce Differential Equations with Boundary Value Problems: Modern Methods and Applications, Wiley, 2010
- R. Haberman Elementary applied partial differential equations. 3rd ed., Prentice Hall, 1998

#### ADDITIONAL BIBLIOGRAPHY

- A. N. Tikhonov, A. A. Samarskii Equations of Mathematical Physics, Dover, 1990
- B. M. Budak, A. A. Samarskii, A. N. Tíjonov Problemas de la Física Matemática. 2 vols., MacGraw Hill y también Mir.
- C. C. Lin, L.A. Segel Mathematics applied to deterministic problems in the natural sciences, SIAM (SIAM Classics in Applied Mathematics vol. 1), 1988
- D.G. Zill Ecuaciones diferenciales con aplicaciones de modelado, 6a. ed., Grupo editorial Iberoamérica.
- G. Strang Introduction to Applied Mathematics, Wellesley-Cambridge Press, 1986
- G.F. Simmons Ecuaciones diferenciales. 2a. ed., McGraw Hill, 1993

- H. F. Weinberger Ecuaciones diferenciales en derivadas parciales, Reverté, 1992
- K. Bryan Differential Equations: A Toolbox for Modeling the World, SIMIODE, 2020
- M. S. Gockenbach Partial differential equations. Analytical and numerical methods, SIAM, 2002
- R. K. Nagle, E. B. Saff, A. D. Snider. Fundamentals of Differential Equations and Boundary Value Problems., Pearson, 2018
- R. L. Burden, J. D. Faires Análisis numérico. 6a. ed. Int., Thomson, 1998
- S. G. Krantz Differential Equations: Theory, Technique and Practice, Chapman and Hall/CRC Press, 2015