# uc3m Universidad Carlos III de Madrid

# Statistical Inference

Academic Year: (2023 / 2024) Review date: 06-09-2023

Department assigned to the subject: Statistics Department Coordinating teacher: GARCIA PORTUGUES, EDUARDO

Type: Compulsory ECTS Credits: 3.0

Year: 1 Semester: 1

#### **OBJECTIVES**

- \* Basic competences
- CB6: Possess and understand the knowledge that provides a basis or opportunity to be original in the development and/or application of ideas, often in a research context.
- CB9: Communicate conclusions, as well as the knowledge and the ultimate reasons that support them, to specialized and non-specialized audiences in a clear and unequivocal manner.
  - CB10: Develop the learning skills that enable further study in a manner that is largely self-directed or autonomous.
- \* General competences
- CG1: Apply the techniques of analysis and representation of information, to adapt it to real problems.
- CG4: Synthesize the conclusions obtained from data analysis and present them clearly and convincingly in a bilingual environment (Spanish and English), both written and oral.
- CG5: Generate new ideas (creativity) and anticipate new situations, in the contexts of data analysis and decision making.
  - CG6: Apply social skills for teamwork and to relate with others in an autonomous way.
- \* Specific competences
- CE1: Apply advanced knowledge of statistical inference in the development of methods for the analysis of real problems.
  - CE2: Use free software such as R and Python for the implementation of statistical analysis.
- CE5: Apply advanced statistical fundamentals for the development and analysis of real problems involving the prediction of a variable response.
- CE6: Apply nonparametric models for the interpretation and prediction of random phenomena.
- CE10: Apply statistical modeling in the treatment of relevant problems in the scientific field.
- \* Learning outcomes
- 1. Understand the fundamental concepts of point estimation, including the role of sampling distributions under normal populations and the Central Limit Theorem.
- 2. Explore different types of estimators and their properties, such as unbiasedness, invariance, consistency, efficiency, and robustness.
- 3. Learn various estimation methods, including the moments method and maximum likelihood method.
- 4. Develop the ability to construct confidence intervals using different techniques, including normal, asymptotic, and bootstrap approaches.
- 5. Develop the ability to construct and understand hypothesis tests using different techniques, including normal and asymptotic theory.

### **DESCRIPTION OF CONTENTS: PROGRAMME**

- 1. Preliminaries
- 1.1. Probability review
- 1.2. Random variables
- 1.3. Random vectors
- 1.4. Transformations of random vectors
- 2. Introduction to statistical inference
- 2.1. Basic definitions

- 2.2. Sampling distributions in normal populations
- 2.3. The central limit theorem
- 3. Point estimation
- 3.1. Unbiased estimators
- 3.2. Invariant estimators
- 3.3. Consistent estimators
- 3.4. Sufficient statistics
- 3.5. Minimal sufficient statistics
- 3.6. Efficient estimators
- 3.7. Robust estimators
- 4. Estimation methods
- 4.1. Methods of moments
- 4.2. Maximum likelihood
- 5. Confidence intervals
- 5.1. The pivotal quantity method
- 5.2. Confidence intervals on a normal population
- 5.3. Confidence intervals on two normal populations
- 5.3. Asymptotic confidence intervals
- 5.4. Bootstrap-based confidence intervals
- 6. Hypothesis tests
- 6.1. Introduction
- 6.2. Tests on a normal population
- 6.3. Tests on two normal populations
- 6.4. Asymptotic tests
- 6.5. p-value of a test
- 6.6. Power of a test and Neyman-Pearson's Lemma
- 6.7. The likelihood ratio test

The program is subject to modifications due to the course development and/or academic calendar.

# LEARNING ACTIVITIES AND METHODOLOGY

The lessons consist of a mixture of theory (methods description) and practice (implementation and practical usage of methods). The implementation of the methods is done with the statistical language R. Students are expected to bring their own laptops to experience the code during some parts of the lessons.

- \* Training activities
- AF1: Theoretical lesson.
- AF2: Practical lesson.
- AF5: Tutorials.
- AF6: Group work.
- AF7: Individual work.
- AF8: On-site evaluation tests.
- \* Teaching methodologies
- MD1: Class lectures by the professor with the support of computer and audiovisual media, in which the main concepts of the subject are developed and the bibliography is provided to complement the students' learning.
- MD3: Resolution of practical cases, problems, etc. posed by the teacher individually or in groups.
- MD4: Presentation and discussion in class, under the moderation of the professor of topics related to the content of the subject, as well as case studies.
- MD5: Preparation of papers and reports individually or in groups.

# ASSESSMENT SYSTEM

The evaluation in the ordinary call is done entirely by continuous evaluation. This is done by a mixture of:

- (a) two quizzes about key ideas and theoretical concepts;
- (b) practical exercises;
- (c) active participation in lessons and voluntary exercises.

The continuous evaluation grade (in the scale 0-10) is

min(0.5 \* A + 0.5 \* B + 0.10 \* C, 10),

# where

- A (in the scale 0-10) is the weighted grade of the quizzes;
- B (in the scale 0-10) is the grade of the practical exercises;
- C (in the scale 0-10) is the grade corresponding to (c).

Students who have not followed the continuous evaluation may take a final exam in the ordinary call with a value of 60% of the final grade.

The grade in the extraordinary call is established by a quiz and the delivery of a set of practical exercises.

Further details are provided in Aula Global. The evaluation is subject to modifications due to the course development and/or academic calendar.

% end-of-term-examination: 0 % of continuous assessment (assignments, laboratory, practicals...): 100

# **BASIC BIBLIOGRAPHY**

- D. Wackerly, W. Mendenhall and R. L. Scheaffer Mathematical Statistics with Applications, Duxbury, 2007
- G. Casella, R. L. Berger Statistical Inference, Thomson Press, 2006
- S. M. Ross Introduction to Statistics, Prentice Hall, 1989

# ADDITIONAL BIBLIOGRAPHY

- B. Efron, R. J. Tibshirani An introduction to the bootstrap, Springer, 1993
- L. Gonick, W. Smith The Cartoon Guide to Statistics, William Morrow, 1993