uc3m Universidad Carlos III de Madrid

Algebra

Academic Year: (2023 / 2024) Review date: 12-11-2019

Department assigned to the subject: Mathematics Department

Coordinating teacher: SOLER GALAN, EUGENIO

Type: Basic Core ECTS Credits: 6.0

Year: 1 Semester: 1

Branch of knowledge: Engineering and Architecture

OBJECTIVES

The student is expected to know and understand the fundamental concepts of:

- Systems of linear equations
- Matrix and vector algebra.
- Vector subspaces in R^n.

The student is expected to acquire and develop the ability to:

- Discuss the existence and uniqueness of solutions of a system of linear equations
- Solve a consistent system of linear equations
- Carry out basic operations with vectors and matrices
- Determine whether a square matrix is invertible or not, and compute the inverse matrix if it exists
- Determine whether a subset of a vector space is a subspace or not
- Find bases of a vector subspace, and compute change-of-basis matrices
- Compute eigenvalues and eigenvectors of a square matrix
- Determine whether a square matrix is diagonalizable or not
- Obtain an orthonormal basis from an arbitrary basis of a subspace
- Solve least-squares problems
- Determine whether a square matrix is orthogonally diagonalizable or not

DESCRIPTION OF CONTENTS: PROGRAMME

- 1. Complex numbers
 - · Numbers sets
 - · Necessity of complex numbers
 - · Binomial form of a complex number
 - · Graphical representation
 - Operations
 - · Complex conjugate, modulus, argument
 - · Polar form of a complex number
 - · Roots of complex numbers
 - · Exponential of a complex number
 - Solving equations
- 2. Systems of linear equations
 - · Introduction to Linear Equations
 - · Geometrical Interpretation
 - · Existence and Uniqueness
 - · Matrix Notation
 - · Gaussian Elimination
 - · Row Equivalence and Echelon Forms
 - · Solving Linear Systems
 - · Homogeneous Systems
 - · Simultaneous Solving
 - · Systems with parameters
- 3. The vector space Kn
 - Vectors
 - · Linear Subspace
 - · Linear Combinations
 - · Subspace Spanned by Vectors
 - · Column and Row Spaces

- · The Matrix Equation Ax=b
- · Null Space
- · Revisiting Linear Systems
- · Linear Independence
- · Basis for a Linear Subspace
- · Dimension of a Linear Subspace
- · Basis for Col A. Row A and Nul A
- · Rank of a Matrix
- · Coordinate Systems
- · Introduction to Linear Transformations
- 4. Matrix algebra
 - Matrix Operations
 - · Transpose of a Matrix
 - · Conjugate Transpose of a Matrix
 - · Inverse of a Matrix
 - · Partitioned Matrices
 - · Determinants
- 5. Eigenvalues and eigenvectors
 - · Eigenvalues & Eigenvectors
 - · The Characteristic Equation
 - Diagonalization
 - · Change of Basis
 - · Transformations between Linear Subspaces
- 6. Orthogonality
 - · Dot Product and Modulus
 - · Orthogonal Sets
 - · Unitary Matrices
 - · Orthogonal Complement
 - · Orthogonal Projection
 - · The Gram-Schmidt Process
 - · The QR decomposition
 - · Least-Squares Problems
- 7. Normal matrices
 - · Schur Decomposition
 - · Normal Matrices & Unitary Diagonalization
 - · Particular Cases of Normal Matrices

LEARNING ACTIVITIES AND METHODOLOGY

The teaching methodology will include:

- Theoretical lectures in large groups, where knowledge that students should acquire will be presented. The course shedule will be available to students and they are expected to prepare the classes in advance.
- Resolution of exercises by the student, which will serve them as a self-assessment and to acquire the necessary skills.
- Tutorships.

ASSESSMENT SYSTEM

- Continuous evaluation: It corresponds to 45% of the final mark. It consists of 3 exams held along the course to assess the student's progression.
- Final exam: It corresponds to 55% of the final mark. It helps assess the student's general understanding of the subject.

% end-of-term-examination: 55 % of continuous assessment (assignments, laboratory, practicals...): 45

BASIC BIBLIOGRAPHY

- David C. Lay Linear Algebra and its Applications, Pearson, 2011 (4th EDITION!)

ADDITIONAL BIBLIOGRAPHY

- Ben Noble, James W Daniel Applied Linear Algebra, Prentice Hall, 1987
- David Poole Linear Algebra: A Modern Introduction, Cengage Learning, 2010 (3rd Edition)
- Jim DeFranza, Daniel Gagliardi Introduction to Linear Algebra with Applications, McGraw Hill, 2009
- W. Keith Nicholson Linear Algebra with Applications, McGraw Hill, 2009 (6th edition)

BASIC ELECTRONIC RESOURCES

- Kahn Academy . Linear Algebra: https://www.khanacademy.org/math/linear-algebra
- Professor Gilbert Strang . ALGEBRA (MIT OpenCourseWare): http://ocw.mit.edu/courses/mathematics/18-06-linearalgebra-spring-2010/video-lectures/