Foundations of Algebra

Academic Year: (2022 / 2023) **Review date: 09/09/2022 17:42:00**

Department assigned to the subject:

Coordinating teacher: ORTEGA GARCIA, ALEJANDRO

Type: Compulsory ECTS Credits : 6.0

Year : 1 Semester : 1

REQUIREMENTS (SUBJECTS THAT ARE ASSUMED TO BE KNOWN)

Basic mathematics, including:

- Solution of linear systems of equations with 2 and 3 unknowns.
- Roots of polynomials. Formula for the roots of quadratic polynomials.
- The graph of a linear function.

OBJECTIVES

The student will acquire the following skills:
1- Basic skills on mathematical reasoning including:
- Distinguish between implications and equivalence.
- Get familiar with the basic methods of mathematical proving like the method by contradiction or the method of mathematical induction.
- Learn to prove set inclusions and set identities.
2- Recognize linear and affine functions and plot them.
3- Handle and simplify polynomial equations and know the basic methods of polynomial root-finding.
4- To know the meaning of the basic trigonometric ratios.
5- To know the ratios of the remarkable angles (0º, 30º, 45º, 60º y 90º).
6- Relate the trigonometric ratios of an arbitrary angle with the ones of an angle between 0 and pi radians.
7- Relate the trigonometric ratios of complementary and supplementary angles.
8- Solve triangules.
9- Determine whether a given trigonometric identity is true or not.
10- Plot elementary trigonometric functions.
11- Identify and plot complex numbers.
12- Operate with complex numbers.
13- Obtain all different representations of a complex number (binary form, polar form, exponential form).
14- Obtain all n nth-roots of a complex number and plot them.
15- To know the Fundamental theorem of Algebra.
16- Compute the Row Reduced Echelon form of a matrix.
17- Solve linear systems using Gaussian elimination.
18- Obtain the matrix representation of a linear system.
19- Perform arithmetic operations with matrices.
20- Obtain the vector expression of a linear system.
21- Relate elementary row operations on a matrix with left products by elementary matrices.
22- Determine whether a given matrix of low size is invertible or not. In the affirmative case, compute its inverse using the algorithm related to the row reduced echelon form of the matrix.
23- Apply the recursive definition of the determinant for low-dimensional matrices.
24- Relate the determinant of a product of matrices with the determinants of each of the factors.
25- To know how the determinant changes when applying elementary row and column operations to the matrix.
26- Obtain the determinant of a matrix through an echelon form of the matrix.
27- Operate with vectors in R^n.
28- Relate the linear independence of a set of vectors with the solution of linear systems.

29- Determine whether a given small set of vectors is linearly independent or not.
30- To know the notion of spanning set and subspace spanned by a set of vectors.
31- Become familiar with the notion of basis of a subspace spanned by a set of vectors.
32- Become familiar with the notion of column space of a matrix.
33- Become familiar with the notion of null-space of a matrix and relate it with the solution of linear systems.

DESCRIPTION OF CONTENTS: PROGRAMME

WEEK 1: Basic Elements
Mathematical notation
Basic methods of mathematical proof (Proof by contradiction, the induction method; equation writing)
Linear and affine functions
Polynomials and polynomial equations

WEEK 2: Trigonometry
Trigonometric ratios in the unit circle
Relationship between complementary and supplementary angles
Relationships between trigonometric ratios. Trigonometric identities
Trigonometric functions

WEEKS 3-4: Complex numbers
Definition. Binomial form
Geometrical representation in R2
Polar and exponential representations. Relation between representations (binomial, polar and exponential)
Operations with complex numbers
Powers of complex numbers
Roots of complex numbers. Geometrical representation
The fundamental theorem of algebra

WEEKS 5-6: Matrices
Basic definitions
Matrix operations
Matrix representation of a linear system
Vector representation of a linear system
Elementary matrices and elementary row operations. Relation to linear systems
Inverse of a matrix

WEEK 7: Determinants
Definition
Determinant of a triangular matrix
Determinant of the product of square matrices
Determinant of the transpose of a matrix
Elementary row operations in a determinant
Determinant of a matrix using the echelon form of a matrix

WEEKS 8-9: Linear systems
Solving linear systems with 2 unknowns. Geometrical representation.
Solving linear systems with 3 unknowns. Geometrical representation.
Solving linear systems with n unknowns. Geometrical representation.
Gaussian elimination
Basic definitions
Echelon and reduced echelon forms
homogeneous systems
Existence and uniqueness theorem
Solutions in parametric form

WEEK 10: Vectors in Rn.
Basic operations
Linear independence
Spanning sets. Span of a set of vectors

Bases
Column space of a matrix
Null space of a matrix
Revisiting linear systems

LEARNING ACTIVITIES AND METHODOLOGY

Discussion of problems in class:
The student is expected to: share own ideas and acquire new ideas coming from the perception of theory and/or exercises by other students.
This activity emphasizes the importance of the role of collaboration as well as dissemination in scientific progress.
This activity is in line with the syllabus to the extent that it allows the student to acquire new ideas that allow a better understanding of the theoretical knowledge explained in class.
Realization of 3 homework assignments to work at home and deliver/present in class. The homework will consist of a series of exercises of varying difficulty.
The objective of this activity is twofold. On the one hand, it is expected that the student maintains a certain daily routine with the work related to the subject. On the other hand, if the difficulty of the exercise requires going deeper into the theory explained in class, the student is expected to acquire and promote the ability to search for resources while acquiring a deeper knowledge of the theory.
This activity emphasizes the value of continuous and personal work in acquiring first and then using the techniques explained in class. Ask as the value the ubiquity, depth and scope of the theory explained in class through the work of more complex problems.
This assignment fits the syllabus to the extent that it allows and encourages the student to deepen the knowledge shared in class.

ASSESSMENT SYSTEM

The final grade will come from:

Continuous Assessment (50%):
Home assignments (30%)
Midterm Exams (20%)
Final Exam(50%)

The continuous assessment will consist of:

A series of homework assigned to the student to work on at home and hand in/present in class.
Throughout the course there will be 2 written mid-term exams during class time.
A final exam to evaluate the acquisition of knowledge by the students.

BASIC BIBLIOGRAPHY

- D. C. Lay, S. R. Lay, J. J. McDonald Linear Algebra and Its Applications, Pearsons, 2015

- Juan de Burgos (Burgos Roma¿n) Curso de a¿lgebra y geometri¿a., Madrid : Alhambra, 1989 ISBN 8420503819

- Richard N. Aufmann Vernon C Barker Richard D Nation College algebra and trigonometry, Boston etc. : Houghton Mifflin, 1997

- Stitz, Carl ; Zeager, Jeff College trigonometry , Open Textbook Library (Corporate Author) Ohio: Stitz Zeager Open Source Mathematics , 2013

- Vicente Bargueño Fariñas Problemas de a¿lgebra con indicaciones teo¿ricas., Madrid : Universidad Nacional de Educacio¿n a Distancia, 1994

ADDITIONAL BIBLIOGRAPHY

- D. C. Lay, S. R. Lay, J. J. McDonald Linear Algebra and Its Applications, Pearson, 2015

- David Poole Linear Algebra: A modern introduction, Cengage Brooks/Cole, 2015