Foundations of Mathematics

Academic Year: (2022 / 2023) **Review date: 21-06-2022**

Department assigned to the subject:

Coordinating teacher: MARHUENDA HURTADO, FRANCISCO

Type: Compulsory ECTS Credits : 6.0

Year : 1 Semester : 1

OBJECTIVES

In this course the students will learn how to use the mathematical tools needed that will enable them to succeed in the successive quantitative subjects, i.e. those in which mathematics models play a key role in developing the key ideas.

The course will promote a positive attitude towards Mathematics both as a subsidiary tool for other successive subjects and as a science in itself, organized around logical deductive conclusions where precision and ingenuity are combined in addressing the social issues.

By solving problems and exercises the students will master the skills and grasp the concepts, integrating both in a meaningful way.

The student after completing this course will be able to define mathematically regions of the real line and the Euclidean plane, work with algebraic expressions and solve several kinds of elementary equations. He will also become familiar with the most important elementary functions used in Social Sciences and will be able to use the tools of differential and integral calculus in one variable.

DESCRIPTION OF CONTENTS: PROGRAMME

The Real Numbers and the Cartesian Plane.
- Notation for the logic structure of Mathematics: Quantifiers, Implications and Equivalence.
- The real numbers. The real line.
- Absolute value and distance on the real line. Intervals: segments and rays. Intersection and union of sets. Inequalities.
- Points, distance and the midpoint formula.
- Equations. Straight lines. Slope of line. General equation of a line. Slope-point equation. Equation of a line determined by two points.
- Circles and Intersections.
- Linear system of equations with two unknowns.
- Gauss elimination method for linear systems.

- Regions defined in the plane by linear inequalities and system of inequalities with two unknowns. Solution regions. Geometric interpretation.

Polynomials and Rational expressions.
- Operations with polynomials. Special binomial products.
- Quadratic equations. Parabolas. Biquadratic equations.
- Roots of polynomials. Factoring polynomials. Synthetic division. Integer roots of polynomials. Ruffini¿s rule.
- Rational expressions. Operations.Equations and System of Equations.

Functions, properties and basic functions.
- Concept of a function.
- Domain and range of a function. The graph of a function.
- Inverse function.
- Composite function.
- Linear functions.

- Radical functions.
- Piecewise defined functions.
- Function transformation. Translations, dilations and symmetry. The absolute value of a function.

Exponential, logarithmic and trigonometric functions.
- Exponential functions.
- Logarithm functions.
- Trigonometrically functions.
- Radical equations.
- Exponential and logarithm equations.

Limits of Functions. Continuity.
- Continuity. Types of discontinuities.
- Limits of a function at a point. Continuity.
- Finding limits of a function at a point.
- Infinite limits.
- Asymptotes. Vertical, horizontal and slant (oblique) asymptotes.
- Rational, exponential and logarithmic asymptotes.
- Continuity. Types of discontinuity. Intermediate value Theorem.

Differentiation
- The derivative of a function. Tangent lines. Instantaneous rate of change.
- The derivative function.
- Rules for differentiation. The chain rule. Implicit differentiation.
- Application of the derivative. Growth of a function. Extreme points of a function. Weierstrass¿ Theorem. Optimization problems.
- Rolle¿s Theorem. Mean value Theorem. L¿Hôpital¿s rule.
- Taylor polynomial.
- Concavity and convexity.
- Applications: Curve sketching of polynomial and rational functions.

Integration
- Antiderivatives. Finding antiderivatives. Basic integration rules.
- Definite integral. The fundamental Theorem of Calculus.
-Techniques of integration: Integration by substitution. Integration by parts. Simple fractions.
- Applications: Area between two curves.

LEARNING ACTIVITIES AND METHODOLOGY

A real-life problem will be posed, which after being expressed in mathematical language will be discussed in a group and the solution will be presented to the rest of the class.

ASSESSMENT SYSTEM

1- There will be two midterm exams in the course. Each midterm will count 1.5 points towards the final grade
2- Students will have to solve a problem proposed by the teacher in a group and present the solution to the rest of the class. This exercise constitutes 1 point of the final grade.
3- There will a final exam. The final exam will count 6 points in the final grade.

BASIC BIBLIOGRAPHY

- James Stewart , Lothar Redlin, Saleem Watson. Calculus: Early transcendentals, Cengage Learning, 2016

- Knut Sydsaeter and Peter J. Hammond Mathematics for Economic Analysis, Pearson.

- Ron Larson, Robert P. Hostetler, Bruce H. Edwards Calculus, Cengage.