Universidad Carlos III de Madrid

Linear Algebra

Academic Year: (2022 / 2023)   **Review date: 29-07-2022**

Department assigned to the subject: Mathematics Department

Coordinating teacher: RASCON DIAZ, CARLOS

Type: Basic Core ECTS Credits : 6.0

Year : 1 Semester : 1

Branch of knowledge: Engineering and Architecture

REQUIREMENTS (SUBJECTS THAT ARE ASSUMED TO BE KNOWN)

Mathematics in scientific-technological "bachillerato" (high school)

OBJECTIVES

The student will become familiar with the concepts of:

1- Linear systems.
2- The algebra of matrices and vectors.
3- Vector subspaces in Rn.
4- Complex numbers.

The student will acquire the skills to be able:

1- Calculate the solution of a system of linear equations
2- Discuss the existence and uniqueness of solutions of a system of linear equations
3- Operate with vectors and matrices
4- Calculate the inverse of a matrix
5- Calculate bases of vector subspaces
6- Calculate eigenvalues and eigenvectors of a matrix
7- Calculate an orthonormal base from any basis
8- Solve least-squares problems
9- Calculate a unitary diagonalization of a normal matrix

DESCRIPTION OF CONTENTS: PROGRAMME

1. Complex numbers
    · Numbers sets
    · Necessity of complex numbers
    · Binomial form of a complex number
    · Graphical representation
    · Operations
    · Complex conjugate, modulus, argument
    · Polar form of a complex number
    · Roots of complex numbers
    · Exponential of a complex number
    · Solving equations
2. Systems of linear equations
    · Introduction to Linear Equations
    · Geometrical Interpretation
    · Existence and Uniqueness
    · Matrix Notation
    · Gaussian Elimination
    · Row Equivalence and Echelon Forms
    · Solving Linear Systems
    · Homogeneous Systems
    · Simultaneous Solving
    · Systems with parameters
3. The vector space Cn
    · Vectors
    · Linear Subspace

- · Linear Combinations
- · Subspace Spanned by Vectors
- · Column and Row Spaces
- · The Matrix Equation Ax=b
- · Null Space
- · Revisiting Linear Systems
- · Linear Independence
- · Basis for a Linear Subspace
- · Dimension of a Linear Subspace
- · Basis for Col A, Row A and Nul A
- · Rank of a Matrix
- · Coordinate Systems
- · Introduction to Linear Transformations

4. Matrix algebra
- · Matrix Operations
- · Transpose of a Matrix
- · Conjugate Transpose of a Matrix
- · Inverse of a Matrix
- · Partitioned Matrices
- · Determinants

5. Eigenvalues and eigenvectors
- · Eigenvalues & Eigenvectors
- · The Characteristic Equation
- · Diagonalization
- · Change of Basis
- · Transformations between Linear Subspaces
- · Abstract Vector Spaces

6. Orthogonality
- · Dot Product and Modulus
- · Orthogonal Sets
- · Unitary Matrices
- · Orthogonal Complement
- · Orthogonal Projection
- · The Gram-Schmidt Process
- · The QR decomposition
- · Least-Squares Problems

7. Normal matrices
- · Schur Decomposition
- · Normal Matrices & Unitary Diagonalization
- · Particular Cases of Normal Matrices

LEARNING ACTIVITIES AND METHODOLOGY

The teaching methodology will include
- Theory classes, where the knowledge that students must acquire will be presented. A textbook (Linear Algebra and its Applications, by David C. Lay) will be followed to facilitate its development. Students will receive the course syllabus and are expected to prepare classes in advance.
- Resolution of exercises by the student that will serve as self-evaluation and to acquire the necessary skills.
- Problem classes, in which the problems proposed to the students are developed and discussed.
- The teacher may pose problems and work to solve individually or in group.
- The teacher will set his schedule of individual tutorials.

ASSESSMENT SYSTEM

- Continuous evaluation: It corresponds to 40% of the final mark. At the beginning of the course, the theory teacher will choose one of these:
1) Mid-term exams held along the course to assess the student's progression.
2) Students must hand in the proposed problems, in the problem class.

- Final exam: It corresponds to 60% of the final mark. It helps assess the student's general understanding of the subject.

ATTENTION: To pass the subject, the student MUST pass the final exam.

% end-of-term-examination: 60

% of continuous assessment (assigments, laboratory, practicals…): 40

BASIC BIBLIOGRAPHY

  - D.C. Lay, S.R. Lay and J.J MacDonald Linear algebra and its applications, Pearson, 2016

  - G. Strang Introduction to Linear Algebra, Cambridge, 2016

  - S.A. García and R.A. Horn A second course in linear algebra, Cambridge, 2017

  - Sergei Treil Linear Algebra Done Wrong, Edited by the author, available from https://www.math.brown.edu/~treil/papers/LADW/LADW.html, 2017 (last update)

ADDITIONAL BIBLIOGRAPHY

  - B. Noble and J. W. Daniel Applied Linear Algebra, 3rd ed., Prentice Hall, 1988

BASIC ELECTRONIC RESOURCES

  - Sergei Treil . Linear Algebra done wrong: https://www.math.brown.edu/~treil/papers/LADW/LADW.html