Numerical Calculus

Academic Year: (2022 / 2023) **Review date: 10/04/2022 18:35:22**

Department assigned to the subject: Mathematics Department

Coordinating teacher: SECO FORSNACKE, DANIEL

Type: Compulsory ECTS Credits : 6.0

Year : 2 Semester :

REQUIREMENTS (SUBJECTS THAT ARE ASSUMED TO BE KNOWN)

Linear Algebra, 1st semester 1st year,
Differential Calculus, 1st semester 1st year,
Integral Calculus, 2nd semester 1st year,
Programming, 1st semester 1st year

OBJECTIVES

Familiarizing with the basic concepts of numerical analysis: algorithms, stability, accuracy, and efficiency.
Interpolating data with different techniques: Lagrange, Hermite, piecewise, splines.
Calculating numerical approximations, choosing the most adequate algorithm for each application, in each of the following problems: quadrature and derivation, systems of linear and non-linear equations, linear least-squares.
Programming the studied algorithms and use other ready-made algorithms, available in MATLAB or other recognized software packages.
Relating real problems and their mathematical models.

DESCRIPTION OF CONTENTS: PROGRAMME

1. Introduction: errors, algorithms and estimates
Sources of error, roundoff, truncation, propagation. Machine numbers, floating-point arithmetics. Taylor polynomials and error. Estimating and bounding errors. Optimal step. Interval arithmetics.

2. Nonlinear equations and nonlinear systems
Nonlinear equations: Mean-value theorem, number of roots in an interval. Bisection, Secant, Newton-Raphson. Fixed-point methods. Convergence order. Error analysis. Nonlinear systems. Accelerated, Taylor and interpolation methods.

3. Methods for linear systems of equations
Linear systems, stability: condition number. Triangular systems. Gaussian elimination. Pivoting. Computing determinants and matrix inverses. Orthogonalization methods and improved methods. Least-squares problems. Regression. Normal equations and QR method. Overdetermined systems. Fast Fourier Transform. Applications.

4. Polynomial interpolation: Lagrange, Hermite, piecewise, splines
Newton/Lagrange Interpolation, errors. Equispaced (or not) nodes. Runge's phenomenon. Hermite interpolation. Richardson's extrapolation. Splines. Natural cubic splines.

5. Numerical quadrature and differentiation
Numerical differentiation: back/forward, central, general, higher order. Errors. Numerical Integration: Newton-Côtes formulae. Errors. Adaptive integration.

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LEARNING ACTIVITIES AND METHODOLOGY

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THEORETICAL-PRACTICAL CLASSES. [44 hours with 100% classroom instruction, 1.67 ECTS]
Knowledge and concepts students must acquire. Student receive course notes and will have basic reference texts to facilitate following the classes and carrying out follow up work. Students partake in exercises to resolve practical problems and participate in workshops and evaluation tests, all geared towards acquiring the necessary capabilities.

TUTORING SESSIONS. [4 hours of tutoring with 100% on-site attendance, 0.15 ECTS]
Individualized attendance (individual tutoring) or in-group (group tutoring) for students with a teacher.

STUDENT INDIVIDUAL WORK OR GROUP WORK [98 hours with 0 % on-site, 3.72 ECTS]

WORKSHOPS AND LABORATORY SESSIONS [8 hours with 100% on site, 0.3 ECTS]

FINAL EXAM. [4 hours with 100% on site, 0.15 ECTS]
Global assessment of knowledge, skills and capacities acquired throughout the course.

METHODOLOGIES

THEORY CLASS. Classroom presentations by the teacher with IT and audiovisual support in which the subject's main concepts are developed, while providing material and bibliography to complement student learning.

PRACTICAL CLASS. Resolution of practical cases and problem, posed by the teacher, and carried out individually or in a group.

TUTORING SESSIONS. Individualized attendance (individual tutoring sessions) or in-group (group tutoring sessions) for students with a teacher as tutor.

LABORATORY PRACTICAL SESSIONS. Applied/experimental learning/teaching in

workshops and laboratories under the tutor's supervision.

ASSESSMENT SYSTEM

% end-of-term-examination/test:	40
% of continuous assessment (assigments, laboratory, practicals…):	60

EVALUATION SYSTEMS

SE1 - FINAL EXAM. [40 %]
Global assessment of knowledge, skills and capacities acquired throughout the course.

SE2 - CONTINUOUS EVALUATION. [60 %]
Assesses papers, projects, class presentations, debates, exercises, internships and workshops throughout the course.

Given that the subject has a highly practical content, a heavier weight (60%) will be assigned to the continuous evaluation. The final grade will be obtained from:
3 practical assignments making up 36% of the grade (12% each);
1 test for 24% of the grade;
1 final exam, summing 40% of the grade.

BASIC BIBLIOGRAPHY

- [CM] Moler, C. B. Numerical computing with MATLAB, SIAM, 2004

- [KA] Atkinson, K. Elementary Numerical Analysis, John Wiley and Sons, 2004

- [MF] Mathews, J. H., Fink, K. D. Numerical methods using Matlab, 3rd edition, Prentice-Hall, 1998

- [TB] Trefthen, L. N., Bau, D., III Numerical Linear Algebra, SIAM, 1997

- [WS] Wen Shen An Introduction to Numerical Computation, World Scientific, 2016

ADDITIONAL BIBLIOGRAPHY

- Sanz Serna, J. M. Diez lecciones de cálculo numérico, Universidad de Valladolid, 2010

- [ABD] Aubanell, A., Benseny, A., Delshams, A. Útiles básicos de cálculo numérico, Universitat Autònoma de Barcelona, 1993

- [HH] Higham, D., Higham, N. MATLAB guide, 2nd edition, SIAM, 2005

- [QSS] Quarteroni, A., Sacco, R., Saleri, F. Numerical mathematics, Springer, 2007