Differential Equations

Academic Year: (2022 / 2023) **Review date: 04-04-2022**

Department assigned to the subject: Mathematics Department

Coordinating teacher: CUESTA RUIZ, JOSE ANTONIO

Type: Basic Core ECTS Credits : 6.0

Year : 2 Semester : 1

Branch of knowledge: Engineering and Architecture

REQUIREMENTS (SUBJECTS THAT ARE ASSUMED TO BE KNOWN)

Calculus I, Calculus II and Linear Algebra

OBJECTIVES

SPECIFIC LEARNING GOALS (PO a):
- To understand the basic theorems of existence and uniqueness in differential equations, paying special attention to the concept of well-posed model.
- To understand the importance of differential equations in the field of biomedical engineering.
- To understand the concept of linear operators and their relation with the superposition principle for solving differential equations.
- To solve elementary differential equations by standard methods.
- To understand the basic techniques to address nonlinear problems arising in differential equations.
- To know the basic differential equation of mathematical engineering and physics as well as the initial and contour problems they lead to.
- To solve partial differential equations by separation of variables and Fourier analysis.

GENERAL ABILITIES (PO a, g, k):
- To understand the necessity of abstract thinking and formal mathematical proofs.
- To acquire communicative skills in mathematics.
- To acquire the ability to model real-world situations mathematically, with the aim of solving practical problems.
- To improve problem-solving skills.

DESCRIPTION OF CONTENTS: PROGRAMME

I) ORDINARY DIFFERENTIAL EQUATIONS

1. Introduction
   1.1 Mathematical modelling
   1.2 Differential equations and their solutions
   1.3 Initial value problems
   1.4 Continuous dependence

2 First-order Differential Equations
   2.1 Existence, uniqueness and continuous dependence of the solutions
   2.2 Sketching the integral curves
   2.3 Basic resolution methods: separable, linear, exact equations, integrating factors
   2.4 Modelling with first-order differential equations

3 Second-order Linear Differential Equations
   3.1 Introduction
   3.2 Second-order linear differential equations
   3.3 Homogeneous equations
   3.4 Homogeneous equation with constant coefficients
   3.5 Inhomogeneous equation: variation of constants
   3.6 Inhomogeneous equation with constant coefficients

4 Linear Systems of Differential Equations
   4.1 Explicit solutions
   4.2 Homogeneous linear systems in matrix form

4.3 Classification of homogeneous linear systems

5 Nonlinear Systems and Stability
5.1 Autonomous systems
5.2 Autonomous systems in one dimension
5.3 Autonomous systems in two dimensions
5.4 Periodic solutions
5.5 Higher dimensions: Lorenz¿s system

II) PARTIAL DIFFERENTIAL EQUATIONS

6 Introduction to Partial Differential Equations
6.1 Generalities
6.2 Superposition principle
6.3 Equations of mathematical physics
6.4 Initial value and boundary value problems
6.5 Types of problems for Poisson¿s and Laplace¿s equations
6.6 Proofs of uniqueness

7 Method of Separation of Variables
7.1 The idea of the method
7.2 Fourier series
7.3 Separation of variables for the wave equation
7.4 Separation of variables for the Laplace equation

8 Sturm-Liouville Problems
8.1 Motivation
8.2 Lagrange-Green identity and self-adjoint problems
8.3 Eigenvalues and eigenfunctions
8.4 Generalised Fourier series and solutions of PDEs
8.5 Rayleigh¿s quotient and minimisation theorem
8.6 Boundary problems in several variables
8.7 Sturm-Liouville problems in several variables

9 Inhomogeneous Problems
9.1 Removal of inhomogeneous conditions
9.2 Eigenfunction expansions
9.3 Periodically-forced waves: resonance
9.4 Inhomogeneous boundary conditions in higher dimensions

LEARNING ACTIVITIES AND METHODOLOGY

1.- Master classes.
2.- Problem classes.
3.- Partial controls.
4.- Final exam.
5.- Tutorials.

ASSESSMENT SYSTEM

Two mid-term tests, 2 x 20%
Final exam, 60%

% end-of-term-examination:	60
% of continuous assessment (assigments, laboratory, practicals…):	40

BASIC BIBLIOGRAPHY

- Haberman, R. Ecuaciones en derivadas parciales con series de Fourier y problemas de contorno, Prentice Hall, 2003
- Robinson, J. C. An Introduction to Ordinary Differential Equations, Cambridge University Press, 2004
- Simmons, G. F. ; Krantz, S. G. Ecuaciones diferenciales. Teoría, técnica y práctica, McGraw-Hill, 2007

ADDITIONAL BIBLIOGRAPHY

- Brannan, J. R., Boyce, W. E. Differential Equations with Boundary Value Problems: An Introduction to Modern Methods & Applications, Wiley., 2010
- Edwards, C. H., Penney, D. E., Calvis, D. Differential Equations and Boundary Value Problems:

Computing and Modeling, Pearson Education, 2016

- Nagle, R. K., Saff, E. B., Snider, A. D. Fundamentals of differential equations , Pearson Addison-Wesley, 2008, 7th ed.

- Tikhonov, A. N., Samarskii, A. A. Equations of Mathematical Physics, Dover, 1990