uc3m Universidad Carlos III de Madrid

Linear Algebra

Academic Year: (2022 / 2023) Review date: 20-05-2022

Department assigned to the subject: Mathematics Department Coordinating teacher: HERNANDO OTER, PEDRO JOSE

Type: Basic Core ECTS Credits: 6.0

Year: 1 Semester: 1

Branch of knowledge: Engineering and Architecture

OBJECTIVES

The student will acquire appropriate skills for:

- 1. Operating and solving simple equations with complex numbers
- 2. Computing the solution of a linear system of equations
- 3. Deciding the uniqueness and existence of the solution of a linear system of equations
- 4. Working with vectors and matrices
- 5. Computing the inverse of a matrix
- 6. Computing bases of linear subspaces
- 7. Computing eigenvalues and eigenvectors of a matrix
- 8. Computing an orthonormal basis
- 9. Solving least square problems
- 10. Diagonalizing symmetric matrices through orthogonal transformations.

Students that pass this subject successfully will have:

- 1. Knowledge and understanding of the mathematical principles of algebra underlying their branch of engineering;
- 2. The ability to apply their knowledge and understanding to identify, formulate and solve mathematical problems using established methods of algebra;
- 3. The ability to select and use appropriate tools and methods of algebra to solve mathematical problems;
- 4. The ability to combine theory and practice of algebra to solve mathematical problems

DESCRIPTION OF CONTENTS: PROGRAMME

- 1. Complex numbers
 - · Numbers sets
 - · Necessity of complex numbers
 - · Binomial form of a complex number
 - · Graphical representation
 - Operations
 - · Complex conjugate, modulus, argument
 - · Polar form of a complex number
 - · Roots of complex numbers
 - · Exponential of a complex number
 - · Solving equations
- 2. Systems of linear equations
 - · Introduction to Linear Equations
 - · Geometrical Interpretation
 - · Existence and Uniqueness
 - · Matrix Notation
 - · Gaussian Elimination
 - · Row Equivalence and Echelon Forms
 - · Solving Linear Systems
 - · Homogeneous Systems
 - · Simultaneous Solving
 - · Systems with parameters
- 3. The vector space Cn

- Vectors
- · Linear Subspace
- · Linear Combinations
- · Subspace Spanned by Vectors
- · Column and Row Spaces
- · The Matrix Equation Ax=b
- Null Space
- · Revisiting Linear Systems
- · Linear Independence
- · Basis for a Linear Subspace
- · Dimension of a Linear Subspace
- · Basis for Col A, Row A and Nul A
- · Rank of a Matrix
- · Coordinate Systems
- · Introduction to Linear Transformations
- 4. Matrix algebra
 - · Matrix Operations
 - · Transpose of a Matrix
 - · Conjugate Transpose of a Matrix
 - · Inverse of a Matrix
 - · Partitioned Matrices
 - Determinants
- 5. Eigenvalues and eigenvectors
 - · Eigenvalues & Eigenvectors
 - · The Characteristic Equation
 - Diagonalization
 - · Change of Basis
 - Transformations between Linear Subspaces
- 6. Orthogonality
 - · Dot Product and Modulus
 - · Orthogonal Sets
 - · Unitary Matrices
 - · Orthogonal Complement
 - · Orthogonal Projection
 - · The Gram-Schmidt Process
 - · The QR decomposition
 - · Least-Squares Problems
- 7. Normal matrices
 - · Schur Decomposition
 - · Normal Matrices & Unitary Diagonalization
 - · Particular Cases of Normal Matrices

LEARNING ACTIVITIES AND METHODOLOGY

- Theory classes in large groups, which could be given in "flipped classroom" format. In them, basic theoretical knowledge and skills will be presented. A textbook will be followed (Linear Algebra and Its Applications, by David C. Lay). The course chronogram will be available in advance to the students to allow them to prepare the classes in advance.
- Problem solving classes in small groups.
- Solving problems by the student.
- The teacher may propose additional homework to be done either individually or in group.

ASSESSMENT SYSTEM

- Continuous evaluation: It corresponds to 40% of the final mark. At the beginning of the course, the theory teacher will choose one of these:
- 1) Mid-term exams held along the course to assess the student's progression. Some or all of these exams may be online.
- 2) Students must hand in the proposed problems, in the problem class.
- Final exam: It corresponds to 60% of the final mark. It helps assess the student's general understanding of the subject.

ATTENTION: To pass the subject, the student MUST pass the final exam.

% end-of-term-examination:	60
% of continuous assessment (assigments, laboratory, practicals):	40

BASIC BIBLIOGRAPHY

- David C. Lay Linear Algebra and its Applications 4th edition, Pearson, 2012.

ADDITIONAL BIBLIOGRAPHY

- B. Noble and J. W. Daniel Applied Linear Algebra, 3rd Edition, Prentice Hall, 1988.
- G. Strang. Linear Algebra and its applications, 4th ed., Wellesley, Cambridge,, 2006
- L. Spence, A. Insel y S. Friedberg Elementary Linear Algebra. A Matrix Approach, Prentice Hall 2000.