Mathematics for Data Science

Academic Year: (2022 / 2023)

Review date: 24-08-2022

Department assigned to the subject: Mathematics Department

Coordinating teacher: BRANDLE CERQUEIRA, CRISTINA

Type: Compulsory ECTS Credits : 3.0

Year : 1 Semester : 1

#### **OBJECTIVES**

While there are many applied mathematics techniques and concepts that are useful (and used) in Data Science, this course focus on the basics of those based on linear algebra and calculus, as they underlie many of the most important applications and algorithms: Matrix algebra, Matrix decompositions.

Basic competences:

To acquire and understand the knowledges that provide the chance of being original in developing or applying ideas, in particular in a research environment.

The students to acquire the learning skills that allow them to keep learning in a self-oriented and autonomous way.

General competences:

To apply the basic theory, techniques, and tools from information research, including data collect, data storage and data analysis, specially for big data, as a basis to apply and develop such techniques to particular problems.

### DESCRIPTION OF CONTENTS: PROGRAMME

- 1. Linear Systems.
- 1.1 Row Reduced Echelon Form.
- 1.2 Gaussian elimination.
- 1.3 Solution of linear systems.
- 2. Vectors in R^n.
- 2.1 Operations with vectors.
- 2.2 Linear combinations and spanned subspace.
- 2.3 Vector equations.
- 2.4 Linear subspaces.
- 2.5 Column, row, and null space of a matrix.
- 2.6 Linear independence.
- 2.7 Bases.
- 2.8 Dimension of a linear subspace.
- 2.9 Coordinate systems.
- 2.10 Linear maps.
- 3. Matrices.
- 3.1 Matrix operations.
- 3.2 The inverse of a matrix.
- 3.3 The LU factorization.
- 3.4 Partitioned matrices.
- 3.5 Determinants.
- 4. Diagonalization.
- 4.1 Eigenvalues and eigenvectors.
- 4.2 Definition and criteria for diagonalization.
- 4.3 How to obtain a diagonalization.
- 4.4 Diagonalization and change of basis.
- 4.5 An introduction to Markov processes.
- 5. Orthogonality.
- 5.1 The inner product and its consequences.
- 5.2 Basic notions on orthogonality:
- 5.2.1 Orthogonal sets.
- 5.2.2 Orthogonal matrices.
- 5.2.3 Orthogonal complement.

- 5.3 The Orthogonal projection.
- 5.4 The Gram-Schmidt orthogonallzation.
- 5.5 Least squares problems.
- 6. Symmetric Matrices
- 6.1 Diagonalization of symmetric matrices.
- 6.2 Quadratic forms.
- 6.3 The singular value decomposition and some applications.
- 6.4 The pseudoinverse of a matrix.
- 6.5 The condition number of a matrix.
- 6.6 Orthogonal least squares.
- 6.7 Principal component analysis.

# LEARNING ACTIVITIES AND METHODOLOGY

Theoretical classes (lectures) Practical problems that students must solve individually as homework Tutorials

### ASSESSMENT SYSTEM

Continuous assessment: six tests, one per week. Additionally, a particular assignment about image compression, that will be introduced in week 6.

Final Exam (in both, ordinary and extraordinary examinations)

% end-of-term-examination:	50
% of continuous assessment (assigments, laboratory, practicals):	50

### BASIC BIBLIOGRAPHY

- David C. Lay, Steven R. Lay, Judi J. McDonald Linear Algebra and Its Applications, Pearson; 5 edition, 2016

## ADDITIONAL BIBLIOGRAPHY

- Gilbert Strang LINEAR ALGEBRA and learning from Data, Wellesley Cambridge Press, 2019
- W. Keith Nicholson Linear Algebra with Applications, McGraw-Hill, 6th edition, 2009