# uc3m Universidad Carlos III de Madrid

# Linear Algebra

Academic Year: (2022 / 2023) Review date: 04-06-2021

Department assigned to the subject: Mathematics Department Coordinating teacher: MUÑOZ GARCIA, JAVIER MANUEL

Type: Basic Core ECTS Credits: 6.0

Year: 1 Semester: 1

Branch of knowledge: Engineering and Architecture

## REQUIREMENTS (SUBJECTS THAT ARE ASSUMED TO BE KNOWN)

Elementary notions on mathematics, including:

- Solution of linear systems (at least 3 equations)
- Evaluation of functions (in particular, of polynomial functions)
- Vectors in R^2 and R^3. Elementary euclidean geometry in R^2 and R^3 (inner product, euclidean distance, modulus of a vector)
- Basic matrix theory.

#### **OBJECTIVES**

The student will acquire the basic concepts of:

- 1. Complex numbers.
- 2. Linear systems.
- 3. Matrix and vector algebra.
- 4. The determinant of a square matrix.
- 5. Vector subspaces in Rn and other vector spaces.
- 6. Eigenvalues and eigenvectors of square matrices.
- 7. Orthogonality and orthonormality of vectors in R^n.

The student will acquire the skills that enable them:

- 1. To work with complex numbers.
- 2. To decide about the existence and uniqueness of solutions for a system of linear equations.
- 3. To find, in the case when they exist, the solutions of a system of linear equations.
- 4. To work with vectors and matrices.
- 5. To compute, in the case when it exists, the inverse of a square matrix.
- 6. To find bases for a vector space or subspace.
- 7. To compute the eigenvalues and eigenvectors of a square matrix.
- 8. To decide whether a square matrix is diagonalizable or not.
- 9. To obtain an orthonormal basis from an arbitrary basis.
- 10. To solve least-squares problems.
- 11. To orthogonally diagonalize a symmetric matrix.

By the end of this content area, students will be able to have:

- 1. Knowledge and understanding of the mathematical principles of linear algebra underlying Electric Engineering;
- 2. The ability to apply their knowledge and understanding to identify, formulate and solve mathematical problems of linear algebra using established methods;
- 3. The ability to select and use appropriate tools and methods to solve mathematical problems using linear algebra;
- 4. The ability to combine theory and practice to solve mathematical problems of linear algebra.

#### **DESCRIPTION OF CONTENTS: PROGRAMME**

Lecture 0. Introduction to Complex Numbers.

- 0.1. Definition. Sum and Product.
- 0.2. Conjugate, Modulus and Argument.
- 0.3. Complex Exponential.
- 0.4. Powers and Roots of Complex Numbers.

## Lecture 1. Systems of Linear Equations.

- 1.1. Introduction to Systems of Linear Equations.
- 1.2. Row Reduction and Echelon Forms.
- 1.3. Vector Equations.
- 1.4. The Matrix Equation Ax=b.
- 1.5. Solution Sets of Linear Systems.
- 1.6. Linear Independence.
- 1.7. Introduction to Linear Transformations.
- 1.8. The Matrix of a Linear Transformation.

#### Lecture 2. Matrix Algebra.

- 2.1. Matrix Operations.
- 2.2. The Inverse of a Matrix.
- 2.3. Block-Partitioned Matrices.

#### Lecture 3. Determinants.

- 3.1. Introduction to Determinants.
- 3.2. Properties of Determinants.

#### Lecture 4. Vector Spaces.

- 4.1. Vector Spaces and Subspaces.
- 4.2. Null Space and Column Space of a Matrix.
- 4.3. Linearly Independent Sets and Bases.
- 4.4. Coordinate Systems.
- 4.5. The Dimension of a Vector Space.
- 4.6. Rank.
- 4.7. Change of Basis.

## Lecture 5. Eigenvalues and Eigenvectors.

- 5.1. Introduction to Eigenvalues and Eigenvectors.
- 5.2. The Characteristic Equation.
- 5.3. Diagonalization of Square Matrices.

### Lecture 6. Orthogonality and Least Squares.

- 6.1. Inner Product, Norm, and Orthogonality.
- 6.2. Orthogonal Sets.
- 6.3. Orthogonal Projections.
- 6.4. The Gram-Schmidt Method and the QR Factorization.
- 6.5. Least-Squares Problems.

## Lecture 7. Symmetric Matrices.

7.1. Diagonalization of Symmetric Matrices.

## LEARNING ACTIVITIES AND METHODOLOGY

The teaching methodology will include:

- Theory classes in large groups, where basic theoretical knowledge and skills will be presented. To facilitate their development, a textbook (¿Linear Algebra and its Applications¿, by David C. Lay) will be followed closely. The weekly planning of the course will be available to the students, allowing them to prepare the classes in advance.
- Solving exercises by the student, which will serve as self-assessment and to acquire the necessary skills.
- Problem solving classes in small groups, where exercises proposed to students will be explained and discussed.
- Using the electronic resources that the teacher will make available to students in the ¿Aula Global¿ platform.
- Tutorial sessions, individual and voluntary, in which students will have the possibility to consult the teacher their doubts and questions on the subject. The time and place of these sessions will be set by the teacher at the beginning of the course.

## **ASSESSMENT SYSTEM**

- Continuous evaluation: It will be carried out through partial exams, which will test the acquisition by the student of the basic concepts and skills of the subject. Its percentage in the final grade will be 40%. At the beginning of the course, the teacher will inform the students about the number of exams to be performed, as well as their exact dates and contents.
- Optionally, the teacher may propose homework, to be done either individually or in group, as a part of the continuous evaluation. Its percentage in the final grade will be less than or equal to 15%, while the partial exams will then correspond to the remaining percentage until completing the 40% of the

#### continuous evaluation.

- Final exam: It will test the global knowledge and understanding of the subject by the student. Its percentage in the final grade will be 60%.

In order to pass the subject, it will be mandatory to get at least 4 points out of 10 (or the proportional grade depending on the total scoring of the exam) in the final exam.

% end-of-term-examination: 60 % of continuous assessment (assignments, laboratory, practicals...): 40

## **BASIC BIBLIOGRAPHY**

- David C. Lay Linear Algebra and its Applications, 4th ed, Pearson Education, 2012...

# ADDITIONAL BIBLIOGRAPHY

- B. Noble y J.W. Daniel Álgebra lineal aplicada, 3ª Ed, Prentice Hall Hispanoamericana, 1989.
- K. Nicholson Elementary Linear Algebra, Mc Graw Hill, 2003
- L. Spence, A. Insel y S. Friedberg Elementary Linear Algebra. A Matrix Approach, Prentice Hall 2000.