Mathematical Methods II

Academic Year: (2022 / 2023) **Review date: 18-05-2022**

Department assigned to the subject: Mathematics Department

Coordinating teacher: ALVAREZ CAUDEVILLA, PABLO

Type: Compulsory ECTS Credits : 6.0

Year : 2 Semester : 1

OBJECTIVES

SPECIFIC LEARNING GOALS:

The aim of this course is to carry out an introduction to Linear Algebra. More precisely, at the end of the term, the students must be able to

1. Solve and discuss systems of linear equations by Gaussian elimination.
2. Manipulate matrices and vectors (addition; multiplication, computation of inverse matrices and determinants whenever is possible).
3. Decide if a family of vectors are linearly dependent or independent.
4. Determine if a family of vectors spans a vector subspace. If so, find one of its basis.
5. Determine if a transformation is linear or not. If so, rewrite such a transformation in terms of matrices in different basis.
6. Determine if an endomorphism can be diagoanlized or not. Diagonalize endomorphisms whenever is possible.
7. Manipulate the abstract notions of scalar product and norm.
8. Obtain an orthonormal basis from a non-orthogonal basis by means of Gram-Schmidt method.
9. State and solve linear model problems by means of least square problems. Solve such problems with orthogonal projections.
10. Obtain singular value decomposition and Moore-Penrose inverse of matrices.
11. State and solve linear model problems by means of least square problems. Solve such problems with singular value decomposition.

OTHER LEARNING GOALS:

1. Develop the ability to analize and summarize.
2. Model and solve problems.
3. Express mathematical reasoning in oral and written form.

DESCRIPTION OF CONTENTS: PROGRAMME

1. Systems of linear equations.

    1.1. Notion of systems of linear equations.
    1.2. Gaussian elimination.
        1.2.1. Matrix notation.
        1.2.2. Row reduction and echelon form.
    1.3. Homogenous linear systems.
    1.4. Applications.

2. Matrices and determinants.

    2.1. Matrices.
        2.1.1. Operation with matrices.
        2.1.2. The inverse of a matrix.
        2.1.3. Partitioned matrices.
        2.1.4. LU factorization.
    2.2. Determinants.
        2.2.1. Properties.
        2.2.2. Cramer's rule.

3. Real vector spaces.

   3.1. Vector spaces and subspaces.
   3.2. Null spaces and column spaces.
      3.2.1. Linear transformation.
   3.3. Linearly independent sets. Bases.
   3.4. Dimension and rank.
   3.5. Change of basis.

4. Eigenvalues and eigenvectors. Diagonalization.

   4.1. Eigenvalues and eigenvectors.
   4.2. Diagonalization.

5. Scalar product and orthogonality. Least square problems.

   5.1. Inner product, length, orthogonality.
   5.2. Orthogonal projections.
   5.3. The Gram-Schmidt process.
   5.4. The least-squares problem.

6. Singular values and vectors. Pseudoinverse.

   6.1. Symmetric matrices.
   6.2. Singular value decomposition.
   6.3. Moore-Penrose pseudoinverse matrix.
   6.4. Applications to least-squares problem.

LEARNING ACTIVITIES AND METHODOLOGY

The theoretical notions of this course will be mainly introduced in the lectures where the instructor will enquiry the students to participate by answering a set of questions that guides them through the learning process. The students will play a key role to build upon the lesson.

The students are encouraged to work autonomously by reading the main references prior to the class, by understanding motivation and applications of the different units in the main references of this course and/or different resources.

In tutorials, students will collaborate and work in small learning groups to solve different problems provided by the instructor along the course.

At the end of every unit the students will have the opportunity to evaluate his understanding by doing a test available at Aula Global.

Every 5 weeks there will be a written exam to evaluate the learning process of each student. These exams will take place in the class. Exceptionally, they might become take-home exams. The instructor will correct and comment the exams within one week after the exam has taken place.

According to students' needs there will one to four extra group tutorials, that are not compulsory. Moreover, there will be two-hour office hours per week.

ASSESSMENT SYSTEM

Roughly every 5 weeks there will be a written exam, up to a total number of three (40%). There will a final exam of the course in January (60%).

% end-of-term-examination:	60
% of continuous assessment (assigments, laboratory, practicals…):	40

BASIC BIBLIOGRAPHY

- B. NOBLE, J. W. DANIEL "Applied linear algebra", Prentice Hall, 3rd edition, 1987
- D. C. LAY "Linear Algebra and Its Applications", Addison Wesley; 3 edition (2002), 2002
- L. N. TREFETHEN, D. BAU "Numerical linear algebra", Society for Industrial and Applied Mathematics, 1997

ADDITIONAL BIBLIOGRAPHY

- C. D. MEYER "Matrix analysis and applied linear algebra", Society for Industrial and Applied Mathematics, 2000