Special Functions and Orthogonal Polynomials

Academic Year: (2021 / 2022) **Review date: 02-07-2021**

Department assigned to the subject: Mathematics Department

Coordinating teacher: DEAÑO CABRERA, ALFREDO

Type: Electives ECTS Credits : 3.0

Year : 1 Semester : 2

REQUIREMENTS (SUBJECTS THAT ARE ASSUMED TO BE KNOWN)

Differential equations

Complex variables

OBJECTIVES

Study special functions relevant in applications from Physics to Finance, their properties and examples.

Study methods of asymptotic analysis for differential equations and integrals: method of Laplace, method of stationary phase and method of steepest descent.

Know basic properties (algebraic and analytical) of orthogonal polynomials on the real line, as well as some of their applications.

CB6, CB7, CB8, CB9, CB10
CG2, CG4, CG5, CG6, CG7
CE1, CE2, CE3, CE4, CE8, CE14

DESCRIPTION OF CONTENTS: PROGRAMME

1. Special Functions of Mathematical Physics. Hypergeometric functions.
1.1. Euler's Gamma and Beta functions. Riemann¿s zeta function.
1.2. Hypergeometric functions.
1.3. Integral representations.
2. Asymptotic methods for ODE and integrals. Laplace and saddle point methods. Steepest descent paths.
2.1. Method of Laplace.
2.2. Method of stationary phase.
2.3. Method of steepest descent.
2.4. Liouville-Green transformations of ODEs.
3. Orthogonal polynomials in the real line. Algebraic and analytic properties. Computational problems.
3.1. Motivation and examples.
3.2. Algebraic and analytical properties.
3.3. Computational problems.
4. An introduction to continuous and discrete Painlevé equations.

LEARNING ACTIVITIES AND METHODOLOGY

Learning activities:

- Theoretical lectures
- Exercise sessions
- Tutorials
- Individual or group work

Methodology:

- Exposition in class of the main concepts of the course by the lecturer.
- Critical reading of recommended bibliography to consolidate the main topics of the course, and to complement them in those directions that the students are more interested in.

- Problem solving sessions, individually or in groups.

Two hours per week will be allocated to tutorials, to solve questions related to the syllabus of the course and the proposed exercises.

ASSESSMENT SYSTEM

Partial evaluation during term (40%): exercise solving, individually or in group, or partial exam.
Final (60%): final exam and/or presentation and exposition of projects related to the contents of the course.

% end-of-term-examination:	60
% of continuous assessment (assigments, laboratory, practicals…):	40

BASIC BIBLIOGRAPHY

- M. E. H. Ismail. Classical and Quantum Orthogonal Polynomials in One Variable, Cambridge University Press, 2009

- P. D. Miller. Applied Asymptotic Analysis, Graduate Studies in Mathematics, Volume 75. American Mathematical Society, 2006
- R. Beals, R. Wong. Special Functions and Orthogonal Polynomials. , Cambridge University Press, 2016

ADDITIONAL BIBLIOGRAPHY

- F. W. J. Olver. Asymptotics and special functions, Academic Press, 1974.

- N. Bleistein, R. A. Handelsman. Asymptotic expansions of integrals , Dover Publications, 1986

- N. M. Temme. Special Functions: an Introduction to the Classical Functions of Mathematical Physics, John Wiley and Sons, 1996
- W. Van Assche. Orthogonal polynomials and Painlevé equations, Australian Mathematical Society Lecture Series 27. Cambridge University Press, 2018