Stochastic Equations for Finance and Biology

Academic Year: (2021 / 2022) **Review date: 31-05-2021**

Department assigned to the subject: Department of Mathematics

Coordinating teacher: BERNAL MARTINEZ, FRANCISCO MANUEL

Type: Compulsory ECTS Credits : 3.0

Year : 1 Semester : 2

REQUIREMENTS (SUBJECTS THAT ARE ASSUMED TO BE KNOWN)

Calculus I
Ordinary Differential Equations
Probability

OBJECTIVES

CB6, CB7, CB9, CB10
CG1, CG2, CG3, CG5, CG6
CE1, CE3, CE5, CE6, CE7, CE8, CE9, CE11

Understand the basic aspects of stochastic modelling: discrete time models; descriptions of random motion; Brownian motion, models of Einstein and Langevin
Get acquainted with stochastic processes in continuous time: diffusive processes and Fokker-Planck equation
Grasp the motivation and subtleties behind the definitions of stochastic integrals, as well as the definition and properties of stochastic differential equations.
Get acquainted with Itô's calculus and its relation with partial differential equations
Understand and know how to program the basic numerical methods for stochastic differential equations and Langevin simulations, as well as the arising numerical errors
Know the most paradigmatic applications of stochastic differential equations in finance and biology

Link to document

DESCRIPTION OF CONTENTS: PROGRAMME

Part One: basic theory

1. Stochastic diffusive processes
   1.1 Brownian motion, models of Einstein and Langevin
   1.2 White noise and Wiener process
   1.3 Fokker-Planck equation
2. Itô's calculus
   2.1 Stochastic integral
   2.2 Stochastic differential equation and Itô calculus
   2.3 Properties of stochastic differential equations
   2.4 Relation to partial differential equations: Feynman-Kac formula
3. Numerical methods for stochastic equations.
   3.1 Euler-Maruyama method
   3.2 Higher-order methods
   3.3 Weak and strong convergence of numerical algorithms
   3.4 Extension to bounded diffusions
   3.5 Langevin simulations

Part Two: applications

4. Biochemical kinetics
5. Black-Scholes model; option pricing
6. Stochastic optimal control; Merton's optimal portfolio
7. Biological evolution

LEARNING ACTIVITIES AND METHODOLOGY

Class hours will be devoted to the following supervised learning activities:

* Master classes / teacher presentations, in which the main concepts of the course are developed, that students are expected to learn. In order to facilitate this, students will be provided with class notes. Bibliography is also provided to complement the students' learning and enable them to dive further in those topics more interesting to them.

* Practical classes, in which problems are didactically solved, supervised computer practice is carried out in the computer room, or students publicly present their work. These classes help develop specific skills.

Additionally, there will be 2 office hours devoted to tutoring students, consisting in individualised teaching activities of theoretical and practical type, such that they call for closer supervision of a teacher even though they might be carried out autonomously by the student. Such activities may be, among other: scheduled tutorials, correction of student's work, and student mentoring.

The remaining credits are earmarked for student's self-study or group study without teacher supervision. During this time, the student solves proposed exercises and reads supplementary texts suggested by the teacher, as well as other texts from the subject's syllabus. During the time, the student may use the computer room.

ASSESSMENT SYSTEM

1) Continuous evaluation, consisting of
- Personal working out and delivery of proposed exercises
- Personal working out and delivery of computer codes used for problems to be solved numerically
- Group solution and exposition of projects (if time and the number of enrolled students allow)

2) Final evaluation: the student's overall knowledge and understanding of the subject will be assessed in a written exam. Its weight in the final mark will be 35%.

% end-of-term-examination:	35
% of continuous assessment (assigments, laboratory, practicals…):	65

BASIC BIBLIOGRAPHY

- Bengt Oksendal Stochastic Differential Equations: An Introduction with Applications (5th Edition), Springer-Verlag, 2014
- Lawrence C. Evans An Introduction to Stochastic Differential Equations, AMS American Mathematical Society, 2013
- Peter E. Kloeden, Eckhard Platen Numerical Solution of Stochastic Differential Equations, Springer-Verlag, 1992

ADDITIONAL BIBLIOGRAPHY

- J. L. García-Palacios Introduction to the theory of stochastic processes and Brownian motion problems Lecture notes for a graduate course,, https://arxiv.org/pdf/cond-mat/0701242.pdf, 2004
- Crispin W. Gardiner Handbook of stochastic methods. Vol. 3., Springer, Berlin, 1985
- Linda J.S. Allen An introduction to stochastic processes with applications to biology, CRC Press, 2010
- Nicolaas G. Van Kampen Stochastic processes in physics and chemistry. Vol. 1, Elsevier, 1992