uc3m Universidad Carlos III de Madrid

Calculus I

Academic Year: (2021 / 2022) Review date: 04-06-2021

Department assigned to the subject: Mathematics Department

Coordinating teacher: LLEDO MACAU, FERNANDO

Type: Basic Core ECTS Credits: 6.0

Year: 1 Semester: 1

Branch of knowledge: Engineering and Architecture

OBJECTIVES

- a. To understand the concept of real number and its implications, mainly the concept of limit.
- b. To understand and manipulate series of real numbers.
- c. To identify functions, their dependence on variables and their basic properties (monotony, parity, continuity, differentiability).
- d. To master the basic operations of Calculus: limits, derivatives, integrals and Taylor expansions.
- e. To interpret the derivative as rate of variation of a function, and the integral as an area.
- f. To understand the Taylor polynomial as the best polynomial local approximation for a sufficiently smooth function, and to apply that approximation to simple cases.
- g. To be able to graph simple functions.
- h. To be able to solve simple optimization problems.

DESCRIPTION OF CONTENTS: PROGRAMME

Part I: Real Numbers and Functions

Chapter 1: The Real Line

- 1.1 Ordered Fields
- 1.2 Number Systems
- 1.3 Absolute value, bounds, and intervals

Chapter 4: Real Functions

- 2.1 Definition and basic concepts
- 2.2 Elementary functions
- 2.3 Operations with functions

Part II: Sequences and Series

Chapter 3: Sequences

- 3.1 Sequences of real numbers
- 3.2 Limit of a sequence
- 3.3 Number e
- 3.4 Indeterminacies
- 3.5 Asymptotic comparison of sequences

Chapter 4: Series

- 4.1 Series of real numbers
- 4.2 Series of nonnegative terms
- 4.3 Alternating series
- 4.4 Telescopic series

Part III: Differential Calculus

Chapter 5: Limit of a Function 5.1 Concept and definition

- 5.2 Algebraic properties
- 5.3 Asymptotic comparison of functions

Chapter 6: Continuity

- 6.1 Definition, properties, and continuity of elementary functions
- 6.2 Discontinuities
- 6.3 Continuous functions in closed intervals

Chapter 7: Derivatives

- 7.1 Concept and definition
- 7.2 Algebraic properties
- 7.3 Derivatives and local behaviour

Chapter 8: Taylor expansions

- 8.1 Asymptotic comparison of functions
- 8.2 Taylor¿s polynomial
- 8.3 Calculating limits
- 8.4 Remainder and Taylor is theorem
- 8.5 Taylor series
- 8.6 Numerical approximations
- 8.7 Local behaviour of functions
- 8.8 Function graphing

Part IV: Integral Calculus

Chapter 9: Primitives

- 9.1 Integration by parts
- 9.2 Primitives of rational functions
- 9.3 Change of variable

Chapter 10: Fundamental Theorem of Calculus

- 10.1 Riemann¿s integral
- 10.2 Properties of the integral
- 10.3 Riemann¿s sums
- 10.4 Fundamental theorem of calculus

Chapter 11: Geometric Applications of Integrals

- 11.1 Area of flat figures
- 11.2 Area of flat figures in polar coordinates
- 11.3 Volumes
- 11.4 Length of curves

LEARNING ACTIVITIES AND METHODOLOGY

The methodology will be the usual one for classes in the classroom, writing on the blackboard, with the occasional help of some resources on-line to illustrate some graphic or computational aspects of the course. Also, the classroom notes will be uploaded in Aula Global at the end of each chapter, along with the problem sheets that will be solved and discussed in the small groups.

ASSESSMENT SYSTEM

The final grade will be assigned through the students' performance in two kinds of tests: three in-class tests, with a weight of 40% in the final grade, together with a final exam, amounting to 60% of the final grade. Failing to attend any of the in-class tests implies obtaining 0 points in that test. The weights above, as well as the points obtained in the in-class tests, will be kept for those students who have to take the extraordinary exam.

% end-of-term-examination: 60 % of continuous assessment (assignments, laboratory, practicals...): 40

BASIC BIBLIOGRAPHY

- J. Stewart Single variable calculus: early transcendentals, Brooks-Cole, 1999
- R. Larson, R.P. Hostetler & B.H. Edwards Calculus, Brooks-Cole, 2005

- S.L. Salas, G.J. Etgen & E. Hille Calculus: One and Several Variables, Wiley, 2006

ADDITIONAL BIBLIOGRAPHY

- H. Anton, I.R.L. Bivens and S. Davis Calculus: Early Transcendentals , Wiley, 2012
- J. Stewart and T. Day Biocalculus. Calculus for Life Sciences, Cengage Learning, 2015
- T.M. Apostol Calculus vol. 1, Wiley, 1991