Foundations of Algebra

Academic Year: (2020 / 2021) **Review date: 19-09-2019**

Department assigned to the subject:

Coordinating teacher: BAYONA REVILLA, VICTOR

Type: Compulsory ECTS Credits : 6.0

Year : 1 Semester : 1

STUDENTS ARE EXPECTED TO HAVE COMPLETED

Basic mathematics, including:

- Solution of linear systems of equations with 2 and 3 unknowns.
- Roots of polynomials. Formula for the roots of quadratic polynomials.
- The graph of a linear function.

COMPETENCES AND SKILLS THAT WILL BE ACQUIRED AND LEARNING RESULTS.

The student will acquire the following skills:
1- Basic skills on mathematical reasoning including:
- Distinguish between implications and equivalence.
- Get familiar with the basic methods of mathematical proving like the method by contradiction or the method of mathematical induction.
- Learn to prove set inclusions and set identities.
2- Recognize linear and affine functions and plot them.
3- Handle and simplify polynomial equations and know the basic methods of polynomial root-finding.
4- To know the meaning of the basic trigonometric ratios.
5- To know the ratios of the remarkable angles (0º, 30º, 45º, 60º y 90º).
6- Relate the trigonometric ratios of an arbitrary angle with the ones of an angle between 0 and pi radians.
7- Relate the trigonometric ratios of complementary and supplementary angles.
8- Solve triangules.
9- Determine whether a given trigonometric identity is true or not.
10- Plot elementary trigonometric functions.
11- Identify and plot complex numbers.
12- Operate with complex numbers.
13- Obtain all different representations of a complex number (binary form, polar form, exponential form).
14- Obtain all n nth-roots of a complex number and plot them.
15- To know the Fundamental theorem of Algebra.
16- Compute the Row Reduced Echelon form of a matrix.
17- Solve linear systems using Gaussian elimination.
18- Obtain the matrix representation of a linear system.
19- Perform arithmetic operations with matrices.
20- Obtain the vector expression of a linear system.
21- Relate elementary row operations on a matrix with left products by elementary matrices.
22- Determine whether a given matrix of low size is invertible or not. In the affirmative case, compute its inverse using the algorithm related to the row reduced echelon form of the matrix.
23- Apply the recursive definition of the determinant for low-dimensional matrices.
24- Relate the determinant of a product of matrices with the determinants of each of the factors.
25- To know how the determinant changes when applying elementary row and column operations to the matrix.
26- Obtain the determinant of a matrix through an echelon form of the matrix.
27- Operate with vectors in R^n.
28- Relate the linear independence of a set of vectors with the solution of linear systems.
29- Determine whether a given small set of vectors is linearly independent or not.
30- To know the notion of spanning set and subspace spanned by a set of vectors.
31- Become familiar with the notion of basis of a subspace spanned by a set of vectors.
32- Become familiar with the notion of column space of a matrix.
33- Become familiar with the notion of null-space of a matrix and relate it with the solution of linear systems.

DESCRIPTION OF CONTENTS: PROGRAMME

WEEK 1: Basic Elements
Mathematical notation
Basic methods of mathematical proof (Proof by contradiction, the induction method; equation writing)
Linear and affine functions
Polynomials and polynomial equations

WEEK 2: Trigonometry
Trigonometric ratios in the unit circle
Relationship between complementary and supplementary angles
Relationships between trigonometric ratios. Trigonometric identities
Trigonometric functions

WEEKS 3-4: Complex numbers
Definition. Binomial form
Geometrical representation in R2
Polar and exponential representations. Relation between representations (binomial, polar and exponential)
Operations with complex numbers
Powers of complex numbers
Roots of complex numbers. Geometrical representation
The fundamental theorem of algebra

WEEKS 5-6: Linear systems
Solving linear systems with 2 unknowns. Geometrical representation.
Solving linear systems with 3 unknowns. Geometrical representation.
Solving linear systems with n unknowns. Geometrical representation.
Gaussian elimination
Basic definitions
Echelon and reduced echelon forms
homogeneous systems
Existence and uniqueness theorem
Solutions in parametric form

WEEKS 7-8: Matrices
Basic definitions
Matrix operations
Matrix representation of a linear system
Vector representation of a linear system
Elementary matrices and elementary row operations. Relation to linear systems
Inverse of a matrix

WEEK 9: Determinants
Definition
Determinant of a triangular matrix
Determinant of the product of square matrices
Determinant of the transpose of a matrix
Elementary row operations in a determinant
Determinant of a matrix using the echelon form of a matrix

WEEK 10: Vectors in Rn.
Basic operations
Linear independence
Spanning sets. Span of a set of vectors
Bases
Column space of a matrix
Null space of a matrix
Revisiting linear systems

ASSESSMENT SYSTEM

The final grade will come from:

- Mid-term assessments (50%).
- Final exam (50%).

The mid-term assessment will consist of several mid-term test. Some homework could also be included in this part of the assessment.

BASIC BIBLIOGRAPHY

- Dennis G. Zill, Jacqueline M. Dewar College algebra, Sudbury, MA : Jones & Bartlett Learning, 2012

- Richard N. Aufmann Vernon C Barker Richard D Nation College algebra and trigonometry, Boston etc. : Houghton Mifflin, 1997

- Stitz, Carl ; Zeager, Jeff College trigonometry , Open Textbook Library (Corporate Author) Ohio: Stitz Zeager Open Source Mathematics , 2013

ADDITIONAL BIBLIOGRAPHY

- D. C. Lay, S. R. Lay, J. J. McDonald Linear Algebra and Its Applications, Pearson, 2015

- David Poole Linear Algebra: A modern introduction, Cengage Brooks/Cole, 2015