

Academic Year: ( 2020 / 2021 )

Review date: 09-07-2020

Department assigned to the subject: Department of Mathematics

Coordinating teacher: TORRENTE ORIHUELA, ESTER AURORA

Type: Basic Core ECTS Credits : 6.0

Year : 1 Semester : 1

Branch of knowledge: Engineering and Architecture

**STUDENTS ARE EXPECTED TO HAVE COMPLETED**

Basic knowledge on vectors and Euclidean space of dimension 2 and 3.

Basic knowledge on matrices and determinants.

Basic knowledge on systems of linear equations.

Basic trigonometry.

**COMPETENCES AND SKILLS THAT WILL BE ACQUIRED AND LEARNING RESULTS.**

1. Learning objectives: (PO: a, CGB1)

- To solve systems of linear equations and to interpret the results.
- To understand the concept of algebraic structure.
- To know and understand the notion of vector spaces and their applications.
- To understand the notion of bases and coordinates in a vector space.
- To understand linear transformations and to represent them by matrices.
- To compute the fundamental vector spaces associated to a matrix.
- To understand the concept of eigenvalues and eigenvectors of a matrix, and to know their computation and applications.
- To compute the QR decomposition of a matrix.
- To find an approximate solution to an inconsistent system of linear equation by least-square fitting.
- To obtain the singular value decomposition of a matrix.

2. Specific skills: (PO: a, CGB1)

- To raise the abstraction level.
- To be able to solve real problems using typical linear algebra tools.

3. General skills: (PO: a, CGB1)

- To improve the oral and written communication ability using the language and signs of mathematics properly.
- To be able to model a real situation described with words using mathematical concepts.
- To improve the ability to interpret a mathematical solution and define its limitations and reliability.
- To be able to use mathematical software.

**DESCRIPTION OF CONTENTS: PROGRAMME**

1. Matrices

- Review of definitions and concepts related to matrices.
- Matrix operations.
- Transpose.
- Inverse.
- Determinant.
- Sets induced by a matrix.

2. Systems of linear equations

- Geometric interpretation of linear systems in  $\mathbb{R}^n$ .
- Existence and uniqueness of solutions.
- Matrix methods to solve systems of linear equations.

3. Vector spaces

- Vector spaces.
- Vector subspaces.
- Operations between subspaces.

#### 4. Basis and dimension

- Spanning sets.
- Basis. Dimension.
- Coordinates.

#### 5. Linear transformations

- Definition and properties.
- Operations between linear transformations.

#### 6. Linear transformation and matrices

- Representation of linear transformations using matrices.

#### 7. Change of basis

- Change of basis.
- Normal form of a linear transformation.

#### 8. Eigenvalues and eigenvectors

- Definitions.
- Characteristic polynomial and characteristic equation.
- Diagonalization.

#### 9. Inner product. Orthogonality

- Inner product.
- Length and angle.
- Orthogonal projection.
- Orthogonal complement.

#### 10. Orthogonal basis

- Orthogonal sets and orthogonal bases.
- Gram-Schmidt process.
- QR factorization.

#### 11. The spectral theorem

- Diagonalization of symmetric matrices.
- Spectral decomposition.

#### 12. Geometry of linear transformations

- Reflections.
- Contractions and dilations.
- Rotations.
- Projections.

#### 13. Least squares

- The least squares problem.
- Geometric interpretation.
- Approximation of functions.

#### 14. Pseudoinverse. Singular value decomposition

- Pseudoinverse.
- Singular value decomposition.
- Applications.

### LEARNING ACTIVITIES AND METHODOLOGY

Synchronous lecture sessions (3 credits) (PO: a, CGB1)

During these sessions we will cover the course topics with the aim of using theory to solve problems.

Practicals, working individually and in groups (3 credits) (PO: a, CGB1)

During these sessions we will solve exercises of different levels of difficulty.

### ASSESSMENT SYSTEM

We follow a continuous-assessment system (40%) plus a final exam (60%):

- The continuous-assessment part consists in two mid-term exams contributing with weight 40% to the final mark. They will be held in regular class hours, according to the current regulations. They allow the students to modify their own learning strategies, if necessary.

- The final exam (contributing with weight 60% to the final mark) will be held at the end of the semester, and allows to assess globally the knowledge of the course topics, skills, and capabilities acquired by the students. (PO: a)

In both the mid-term and final exams, competence CBG3 will be evaluated.

There is a resit exam in June for those students who did not obtain the required end-of-semester mark. This resit exam has a maximum mark of 10, and the June final mark is given by  $\max(EE, 0.6 EE + 0.4 EC)$ , where EE (resp. CA) is the resit-exam (resp. continuous-assessment) mark.

<b>% end-of-term-examination:</b>	60
<b>% of continuous assessment (assignments, laboratory, practicals...):</b>	40

#### BASIC BIBLIOGRAPHY

- B. Kolman "Introductory linear algebra: an applied first course", Prentice Hall, 8th ed. - 2005
- D. C. Lay "Linear algebra and its applications", Addison Wesley, 4th ed. - 2011

#### ADDITIONAL BIBLIOGRAPHY

- D. POOLE "Álgebra Lineal. Una introducción moderna", Thomson - Primera edición - 2004.
- O. BRETSCHER "Linear algebra with applications", Prentice Hall, 4th ed. - 2009