

Academic Year: ( 2020 / 2021 )

Review date: 10-07-2020

Department assigned to the subject: Department of Mathematics

Coordinating teacher: MARTINEZ RATON, YURI

Type: Electives ECTS Credits : 6.0

Year : 2 Semester : 2

**STUDENTS ARE EXPECTED TO HAVE COMPLETED**

Requires working knowledge in partial differential equations and stochastic processes. The previous course on Modeling and Simulation of Complex Systems will be particularly useful.

**COMPETENCES AND SKILLS THAT WILL BE ACQUIRED AND LEARNING RESULTS.**

Every year this course will provide one or several advanced topics in complex systems. For this course the topics will be "Extended systems far from equilibrium" and "Modelling biological systems". The goal is to provide the student with the standard techniques to model and describe diverse complex systems, using both continuum and discrete approaches, as well as with a quantitative picture of the current state-of-the-art in the corresponding topics.

**DESCRIPTION OF CONTENTS: PROGRAMME**

Part I: Modeling biological systems.

1. Dynamics of a single species population. Continuum models.
  - A simple model of insect pests.
  - Delayed models. Regular solutions.
  - Models with physiological delays to study diseases.
  - Population models with age distribution.
2. Discrete models of populations of a single species.
  - Simple models and graphical solution procedure.
  - Discrete logistics equation. Chaos.
  - Stability, periodic solutions and forks.
  - Discrete models with delay.
3. Continuous models of dynamics of populations of two species.
  - Lotka-Volterra predator-prey model.
  - Complexity and stability.
  - A realistic predator-prey model.
  - Competitive species. The principle of competitive exclusion.
  - Mutualism or symbiosis.
  - Threshold value phenomenon.
4. Discrete models of two species
  - Predator-prey model.
  - Synchronized emergence of insect pests.
5. Chemical kinetics
  - Enzymatic kinetics.
  - Michaelis-Menten theory.
  - Cooperative phenomena.
  - Autocatalysis, activation and inhibition.
  - Multiple stationary states.
6. Biological oscillators and switches.
  - Feedback control mechanisms.
  - Oscillators and switches involving two or more species.
  - A simple oscillator of two species. Bifurcations to periodic solutions.
7. Belousov-Zhabotinski reaction.

- Field-Noyes model.
- Analysis of stability and limit cycles.
- Non-local stability of the model.
- A simple approach with relaxation oscillations.

Part II: Extended systems far from equilibrium.

1. Introduction.

2. Reaction and diffusion systems:

- Diffusion equation.
- Models for animal dispersal.
- Nonlocal effects and long range diffusion.
- Reaction-diffusion models.

3. Traveling waves in non-linear systems:

- Basics.
- Fisher-Kolmogoroff equation.
- Waves in other systems.
- Waves in excitable media.

4. Pattern formation:

- Introduction.
- Basics of the linear stability analysis.
- Models featuring linear instabilities.
- Basics of the non-linear analysis.
- Non-linear models of pattern formation.
- Amplitude equations.

Practical sessions:

Numerical simulations of interface equations.

Morphology visualization and data analysis.

#### LEARNING ACTIVITIES AND METHODOLOGY

Teaching time will be spent in the following activities:

- Master Classes: The aim is to provide the student with the specific cognitive competence of the subject. The teacher will present the topics of the matter. To ease the study students will be provided the classroom notes and will have access to the basic texts to further investigate those topics they are more interested in.

- Practical Classes: They are devoted to solving problems, working on practical exercises in computer rooms or presenting a topic by the students. These activities are aimed at reaching competence in the specific skills.

Additionally, some time will be spent in tutored activities, like mentoring classes, expositions of works, or guided problem solving.

The remaining time will be devoted to personal study by the students, either by themselves or in group, without teacher supervision, as well as to look for --and study-- the recommended bibliography. During this time students will have free access to the computer rooms.

#### ASSESSMENT SYSTEM

40% of the final grade will be obtained through preparing written documents, and solving and presenting problems in the classroom along the course.

The remaining 60% of the final grade will be obtained through a final exam at the end of the course.

This assessment system could be modified due to exceptional situations.

**% end-of-term-examination:** 60

**% of continuous assessment (assignments, laboratory, practicals...):** 40

#### BASIC BIBLIOGRAPHY

- J. D. Murray Mathematical Biology I and II, Springer, 2002

- M. Cross and H. Greenside Pattern Formation and Dynamics in Nonequilibrium Systems, Cambridge University, 2009

#### ADDITIONAL BIBLIOGRAPHY

- D. Walgraef Spatio-Temporal Pattern Formation, Springer, 1997
- G. Nicolis Introduction to Nonlinear Science, Cambridge University Press, 1995
- I. R. Epstein y J. A. Pojman An Introduction to Nonlinear Chemical Dynamics, Oxford University Press, 1998
- R. Hoyle Pattern Formation: An introduction to Methods, Cambridge University Press, 2006
- S. H. Strogatz Nonlinear Dynamics and Chaos, Perseus Books, 1994
- S. Kinoshita Pattern Formations and Oscillatory Phenomena, Elsevier, 2013
- W. Bialek Biophysics: Searching for Principles, Princeton University Press, 2012