Stochastic differential equations

Academic Year: (2020 / 2021) **Review date: 24/07/2020 16:09:25**

Department assigned to the subject:

Coordinating teacher: MOSCOSO CASTRO, MIGUEL ANGEL

Type: Electives ECTS Credits : 6.0

Year : 2 Semester : 1

REQUIREMENTS (SUBJECTS THAT ARE ASSUMED TO BE KNOWN)

Linear Algebra
Calculus
Ordinary Differential Equations
Probability and Statistics

OBJECTIVES

* To become familiar with the basic stochastic tools in mathematics
* To understand the effects of fluctuations and random variables both analytically and numerically.
* To know the most important applications of stochastic differential equations
* To understand and apply the basic stochastic models in discrete time and characterize their potential stationary states.
* To have knowledge of the general structure of stochastic processes and their mathematical description.
* To understand and use the concepts of random walk and Poisson process as the main ingredients in basic stochastic models.
* To understand and apply the basic models in continuous time, in particular diffusive process and their description as probability models.
* To know and use the stochastic calculus and to understand the concept of stochastic differential equations and their solutions.
* To become familiar with the most relevant applications of stochastic differential equations
* To become familiar with the most basic tools of numerical approximations of stochastic process.

DESCRIPTION OF CONTENTS: PROGRAMME

1. Probability and random variables
1.1 Probability spaces
1.2 Random variables
1.3 Stochastic processes
1.4 Expected value, variance
1.5 Distribution functions
1.6 Independence
1.7 Strong law of large numbers. Central limit theorem
1.8 Martingales
2. Stochastic processes in discrete time
2.1 Introduction
2.2 Markov Processes
2.3 Renewal Processes
2.4 Branching processes
2.5 Queue processes
3. General Stochastic Processes / Basic Applications
3.1 Definition of Stochastic Process
3.2 Trajectories, Markov property.
4. Basic Applications
4.1 Random walk process

4.2 Poisson process.
5. Diffusive processes and Fokker Planck equation
5.1 Definition of diffusion processes
5.2 Backward Kolmogorov and the Fokker-Planck Equations
5.3 Applications and solution methods
6. Ito¿s Calculus
6.1 Stochastic Integrals
6.2 Ito¿s formula
6.3 Applications
7. Stochastic Differential Equations
7.1 Definition and examples
7.2 Existence and uniqueness of solutions
7.3 Properties of the solutions
7.4 Explicit solutions
7.5 Relationship with diffusive processes PDEs
7.6 Feynman-Kac formula
7.7 Solution of PDEs using stochastic differential equations
8. Applications: Mathematical Finance and Stochastic control
8.1 Introduction
8.2 The Black-Scholes model
8.3 Derivatives: futures and options
8.4 Derivative pricing: The Black-Scholes equation
8.5 Generalizations and limitations of the Black-Scholes equation
8.6 Introduction to stochastic control
8.7 Deterministic Control. The Hamilton-Jacobi-Bellman equation
8.8 Stochastic control: the stochastic Hamilton-Jacobi-Bellman equation
8.9 Dynamic programming
8.10 Applications: Optimal portfolio
9. Numerical integration of Stochastic differential Equations
9.1 Introduction
9.2 The Euler-Maruyama method
9.3 Weak and Strong convergence of numerical algorithms
9.4 Higher order methods
9.5 Applications
10. Langevin Simulation
10.1 Microscopic description of the dynamics of a particle
10.2 Fluctuation-dissipation theorem
10.3 Overdamped Langevin equation
10.4 Equilibrium.
10.5 Numerical approximation of Langevin equation.
10.6 Applications to potential and non-potential systems: epidemic spreading and the Ising (or phi4) model

LEARNING ACTIVITIES AND METHODOLOGY

Lecture time will be devoted to the following activities:
a) Theory exposition: with the objective of achieving the specific competences and skills of the subject. During the lectures the student will be exposed to the concepts and knowledge the students have to acquire. To this end, the students will receive the lecture notes and basic reference books in advance, so they will be able to advance and deepen those subjects in which they are more interested.
b) Practice lessons: these lectures will consist on problem solving, hands-on expositions in the computer rooms or students' presentations. These lectures will allow the student to develop the specific competences of the subject.

ASSESSMENT SYSTEM

% end-of-term-examination/test:	50
% of continuous assessment (assigments, laboratory, practicals…):	50

1- Continuous evaluation. There will be homework assignments. They will test the acquisition of basic skills and concepts by the student. Its percentage in the final grade will be 50%.
2- Written exams with a 50% in the final grade.
3- Extra test. Those students who have not passed the subject in the regular period will have the opportunity to do an extra final test. Its percentage in the grade will be 100%, although for those students who did the partial tests in the regular period, I will apply the same rules as in the regular period, whenever this improves the grade of the extra test.

% end-of-term-examination/test:	50
% of continuous assessment (assigments, laboratory, practicals…):	50

BASIC BIBLIOGRAPHY

- Bernt Oksendal Stochastic Differential Equations: An Introduction with Applications, 6th Edition, Springer, 2014

- Lawrence C. Evans An Introduction to Stochastic Differential Equations, AMS, 2013