Advanced methods in matrix analysis

Academic Year: (2020 / 2021) **Review date: 10-09-2020**

Department assigned to the subject: Department of Mathematics

Coordinating teacher: MORO CARREÑO, JULIO

Type: Compulsory ECTS Credits : 6.0

Year : 1 Semester : 1

STUDENTS ARE EXPECTED TO HAVE COMPLETED

Linear Algebra
Calculus

COMPETENCES AND SKILLS THAT WILL BE ACQUIRED AND LEARNING RESULTS.

1- Knowledge, applications, and use of basic matrix factorizations: LU, Cholesky, and QR.
2- Knowledge, applications, and use of canonical forms under similarity: Schur and Jordan forms.
3- Spectral Theory, applications, and use of normal matrices and its particular cases: Hermitian/antihermitian matrices, orthogonal and unitary matrices.
4- The use of Min-max theorem for Hermitian matrices and consequences.
5- Knowledge, applications, and use of the Singular Value Decomposition, in particular in approximation problems.
6- Knowledge, applications, and use of Moore-Penrose pseudoinverses in the solution of least squares problems.
7- Knowledge, applications, and use of matrix norms.
8- Knowledge, applications, and use of basic matrix perturbation results.

DESCRIPTION OF CONTENTS: PROGRAMME

1- Basics on Matrix Analysis.
1.1- Block partitioned matrices and block operations
1.2- Schur complement and properties.
1.3- Projection matrices and orthogonal projection matrices.
1.4- Basics on eigenvalues and eigenvectors. Diagonalizable matrices.

2- Vector and matrix norms.
2.1- Vector norms. Monotone and absolute norms.
2.2- Important vector norms: 1, 2, and infinity.
2.3- Consistent matrix norms. Important examples: 1, 2, infinity, and Frobenius.
2.4- Convergent matrices and spectral radius.

3- LU and QR factorizations.
3.1- LU factorization and Schur complement.
3.2- Unitary matrices.
3.3- QR factorization and the Gram-Schmidt method.

4- Canonical forms under similarity.
4.1- Block diagonalization and matrix Sylvester equations.
4.2- Schur Form.
4.3- Jordan canonical form.

5- Normal Matrices. Hermitian Matrices.
5.1- Spectral theorem of normal matrices.
5.2- Variational characterizations of eigenvalues of Hermitian matrices: the min-max theorem.
5.3- Eigenvalue interlacing in Hermitian matrices.
5.4- Sylvester's inertia theorem for Hermitian matrices.

6- Singular value decomposition and pseudoinverses.
6.1- The theorem of the singular value decomposition.
6.2- Optimal approximation by matrices with smaller rank.
6.3- Pseudoinverses. Moore-Penrose pseudoinverse.
6.4- Applications to least squares problems.

7- Matrix perturbation theory.
7.1- Perturbation of solutions of linear systems: the condition number of a matrix.
7.2- Perturbation of eigenvalues and invariant subspaces: general theory
7.3- Perturbation of eigenvalues and invariant subspaces: the symmetric case.
7.4- Perturbation of singular values and vectors

LEARNING ACTIVITIES AND METHODOLOGY

1- Theory classes delivered by the course instructor: the key theoretical results will be presented.

2- Problem solving by the students on their own.

3- Individual meetings of the instructor with students seeking guidance on the contents of the course.

ASSESSMENT SYSTEM

Tests (40%). Final exam (60%).

% end-of-term-examination:	60
% of continuous assessment (assigments, laboratory, practicals…):	40

BASIC BIBLIOGRAPHY

- G. W. Stewart and J-G. Sun Matrix Perturbation Theory, Academic Press, 1991
- R. A. Horn and C. R. Johnson Matrix Analysis, Cambridge University Press, 1985
- R. A. Horn and C. R. Johnson Topics in Matrix Analysis, Cambridge University Press, 1991

ADDITIONAL BIBLIOGRAPHY

- B. Noble y J.W. Daniel Álgebra Lineal Aplicada, Prentice Hall Hispanoamericana, 1989
- David S. Watkins Fundamentals of Matrix Computations, John Wiley & Sons, 1991
- F. R. Gantmacher The theory of Matrices, Vols 1 and 2,, AMS-Chelsea, 1998 (reprinted from 1959 original edition)
- J. W. Demmel Applied Numerical Linear Algebra, SIAM, 1997
- L. N. Trefethen y D. Bau Numerical Linear Algebra, SIAM, 2000
- Peter Lancaster and Miron Tismenetski The theory of matrices (2nd Edition), Academic Press, 1985