uc3m Universidad Carlos III de Madrid

Linear Algebra

Review date: 27-04-2020 Academic Year: (2019 / 2020)

Department assigned to the subject: Mathematics Department Coordinating teacher: TORRENTE ORIHUELA, ESTER AURORA

Type: Basic Core ECTS Credits: 6.0

Year: 1 Semester: 1

Branch of knowledge: Engineering and Architecture

REQUIREMENTS (SUBJECTS THAT ARE ASSUMED TO BE KNOWN)

Basic knowledge on vectors and Euclidean plane and space.

Basic knowledge on matrices and determinants. Basic knowledge on systems of linear equations.

Basic trigonometry.

OBJECTIVES

- 1. Learning objectives: (PO: a, CGB1)
- To solve systems of linear equations and to interpret the results.
- To understand the notion of vector spaces.
- To understand the notion of bases and coordinates in a vector space.
- To represent a linear transformation by a matrix.
- To compute the image and the kernel of a linear transformation.
- To compute the eigenvalues and the eigenvectors of a matrix.
- To compute the QR decomposition of a matrix.
- To find an approximate solution to an (inconsistent) system of linear equation by least-square fitting.
- To compute the singular value decomposition of a matrix.
- 2. Specific skills: (PO: a, CGB1)
- To raise the abstraction.
- To be able to solve real problems using typical linear algebra tools.
- 3. General skills: (PO: a, CGB1)
- To improve the oral and written communication abilities using mathematics language and signs properly.
- To be able to model a real situation by a linear transformation.
- To improve the ability to interpret a mathematical solution and define its limitations and reliability.
- To be able to use mathematical software.

DESCRIPTION OF CONTENTS: PROGRAMME

- 1 Matrices
- Review of definitions and concepts related to matrices.
- Matrix operations.
- Transpose.
- Inverse.
- Determinant.
- Sets induced by a matrix.
- 2. Systems of linear equations
- Geometric interpretation of linear systems in R^n.
- Existence and uniqueness of solutions.
- Matrix methods to solve systems of linear equations.
- Vector spaces
- Vector spaces.
- Vector subspaces.
- Operations between subspaces.
- 4. Basis and dimension

- Spanning sets.
- Basis. Dimension.
- Coordinates.

5. Linear transformations

- Definition and properties.
- Operations between linear transformations.

6. Linear transformation and matrices

- Representation of linear transformations using matrices.

7. Change of basis

- Change of basis.
- Normal form of a linear transformation.

8. Eigenvalues and eigenvectors

- Definitions.
- Characteristic polynomial and characteristic equation.
- Diagonalization.

9. Inner product. Orthogonality

- Inner product.
- Length and angle.
- Orthogonal projection.
- Orthogonal complement.

10. Orthogonal basis

- Orthogonal sets and orthogonal bases.
- Gram-Schmidt process.
- QR factorization.

11. The spectral theorem

- Diagonalization of symmetric matrices.
- Spectral decomposition.

12. Geometry of linear transformations

- Reflections.
- Contractions and dilations.
- Rotations.
- Projections.

13. Least squares

- The least squares problem.
- Geometric interpretation.
- Approximation of functions.

14. Pseudoinverse. Singular value decomposition

- Pseudoinverse.
- Singular value decomposition.
- Applications.

LEARNING ACTIVITIES AND METHODOLOGY

Lecture sessions (3 credits) (PO: a, CGB1)

Problem sessions working individually and in groups (3 credits) (PO: a, CGB1)

ASSESSMENT SYSTEM

We follow a continuous-assessment system (40%) plus a final exam (60%):

- The continuous-assessment part consists in two mid-term contributing with weight 40% to the final mark. They will be held in regular class hours, according to the current regulations. They allow the students to modify their own learning strategies, if necessary.
- The final exam (contributing with weight 60% to the final mark) will be held at the end of the semester, and allows to assess globally the knowledge of the course topics, skills, and capabilities acquired by the students. (PO: a)

In both the mid-term and final exams, competence CBG3 will be evaluated.

There is a resit exam in June for those students who did not obtain the required end-of-semester mark. This resit exam has a maximum mark of 10, and the June final mark is given by max (EE, 0.6 EE + 0.4 EC), where EE (resp. CA) is the resit-exam (resp. continuous-assessment) mark.

% end-of-term-examination: 60

% of continuous assessment (assignents, laboratory, practicals...):

BASIC BIBLIOGRAPHY

- D. C. LAY "Linear algebra and its applications", Addison-Wesley 4th ed. 2009.
- D. POOLE "Linear algebra: a modern introduction", Thomson 3rd ed. 2010.

ADDITIONAL BIBLIOGRAPHY

- B. KOLMAN "Introductory linear algebra: an applied first course", Prentice Hall, Octava edición 2006
- B. KOLMAN "Álgebra lineal", Prentice Hall Octava edición 2006.
- B. NOBLE, J. W. DANIEL "Álgebra lineal aplicada", Prentice Hall Hispanoamericana Tercera edición 1989.
- O. BRETSCHER "Linear algebra with applications", Prentice Hall 4th ed. 2009...

BASIC ELECTRONIC RESOURCES

- J. Salas, A. Torrente y E.J.S. Villaseñor . Ejercicios de autoevaluación: http://euler.uc3m.es/algebralineal/