Numerical Methods

Academic Year: (2019 / 2020) **Review date: 16-05-2019**

Department assigned to the subject: Mathematics Department

Coordinating teacher: SECO FORSNACKE, DANIEL

Type: Compulsory ECTS Credits : 6.0

Year : 2 Semester : 1

REQUIREMENTS (SUBJECTS THAT ARE ASSUMED TO BE KNOWN)

Linear Algebra, Differential Calculus, Integral Calculus, Programming

OBJECTIVES

To get acquainted with the basic concepts from numerical analysis: algorithms stability, accuracy, and efficiency.
To interpolate data through different techniques: Lagrange, Hermite, piecewise, splines.
To compute numerical approximations, selecting the most appropriate algorithm in each application, for the following problems: quadrature and derivation, linear and nonlinear equations systems, linear least squares.
To program the algorithms studied in the course and to use previously programmed algorithms available, for example, in MATLAB or other widely used software packages.
To map real problems with their mathematical models.

DESCRIPTION OF CONTENTS: PROGRAMME

1. Introduction: errors, algorithms and estimates
Sources of error, roundoff, truncation, propagation. Machine numbers, floating-point arithmetics. Taylor polynomials and error. Estimating and bounding errors. Optimal step. Interval arithmetics.

2. Polynomial interpolation: Lagrange, Hermite, piecewise, splines
Newton/Lagrange Interpolation, errors. Equispaced (or not) nodes. Runge's phenomenon. Hermite interpolation. Richardson's extrapolation. Splines. Natural cubic splines.

3. Numerical quadrature and differentiation
Numerical differentiation: back/forward, central, general, higher order. Errors. Numerical Integration: Newton-Côtes formulae. Errors. Adaptive integration.

4. Direct methods for linear systems of equations
Linear systems, stability: condition number. Triangular systems. Gaussian elimination. Pivoting. Computing determinants and matrix inverses. Conditioning. Orthogonalization methods and improved methods.

5. Nonlinear equations and nonlinear systems
Nonlinear equations: Mean-value theorem, number of roots in an interval. Bisection, Secant, Newton-Raphson. Fixed-point methods. Convergence order. Error analysis. Nonlinear systems. Accelerated, Taylor and interpolation methods.

6. Linear least squares problems
Least-squares, normal equations. Regression. Normal equations and QR method. Overdetermined systems. Applications.

LEARNING ACTIVITIES AND METHODOLOGY

One of the purposes of this course is to provide the mathematical foundations of numerical methods, to analyze and establish their basic theoretical properties (stability, accuracy, computational complexity), and to demonstrate their performances on examples and counterexamples which outline their pros and cons. The primary aim is to develop algorithmic thinking emphasizing on long-living computational concepts. Every chapter is supplied with examples, proofs, exercises, videos and applications of the discussed theory. The course relies throughout on well tested numerical procedures for which we include and discuss codes and test files.

Students should write their own codes by studying and eventually rewriting the codes given by the Teacher in Aula Global. The personal codes should be run, tested and uploaded into Aula Global.
There will be one session per week of theoretical classes, and the other session will sometimes be a practical session where we will study how to solve different problems in a regular room; and laboratory sessions where we will work on implementation and usage of different methods and applications on computers.
In the tutoring sessions, participation will be possible individually or in groups as the students decide and there will be at least 4 hours per week dedicated to this in the weekly schedule. Students are expected to dedicate around 98 hours of personal work outside the class for this subject.

ASSESSMENT SYSTEM

Given that the subject has a highly practical content, a heavier weight (60%) will be assigned to the continuous evaluation. The final grade will be obtained from:
3 practical assignments making up 36% of the grade (12% each);
1 test for 24% of the grade;
1 final exam, summing 40% of the grade.

% end-of-term-examination:	40
% of continuous assessment (assigments, laboratory, practicals…):	60

BASIC BIBLIOGRAPHY

- [CM] Moler, C. B. Numerical computing with MATLAB, SIAM, 2004
- [KA] Atkinson, K. Elementary Numerical Analysis, John Wiley and Sons, 2004
- [MF] Mathews, J. H., Fink, K. D. Numerical methods using Matlab, 3rd edition, Prentice-Hall, 1998
- [TB] Trefthen, L. N., Bau, D., III Numerical Linear Algebra, SIAM, 1997
- [WS] Wen Shen An Introduction to Numerical Computation, World Scientific, 2016

ADDITIONAL BIBLIOGRAPHY

- [ABD] Aubanell, A., Benseny, A., Delshams, A. Útiles básicos de cálculo numérico, Universitat Autònoma de Barcelona, 1993
- [HH] Higham, D., Higham, N. MATLAB guide, 2nd edition, SIAM, 2005
- [QSS] Quarteroni, A., Sacco, R., Saleri, F. Numerical mathematics, Springer, 2007