# uc3m Universidad Carlos III de Madrid

# Mathematics

Academic Year: (2019 / 2020) Review date: 14-06-2018

Department assigned to the subject: Economics Department Coordinating teacher: RINCON ZAPATERO, JUAN PABLO

Type: Compulsory ECTS Credits: 6.0

Year: 1 Semester: 1

# REQUIREMENTS (SUBJECTS THAT ARE ASSUMED TO BE KNOWN)

Intro to Statistics and Mathematics

#### **OBJECTIVES**

**Basic Skills** 

To acquire the knowledge and understanding that provide a basis or opportunity for originality in developing and/or applying ideas, often in a research context.

Students must possess the learning skills that enable them to continue studying in a way that will be largely self-directed or autonomous.

### General Skills

Students can apply advanced mathematical knowledge to economic analysis.

Students can apply advanced knowledge of specific programs of economics, mathematics and econometrics.

# Specific Skills

Students are able to interpret:

- the basic concepts of topology in Euclidean spaces of any dimension and apply them to problems of economic analysis;
- advanced problems of sequences and series of real numbers and apply them to problems of economic analysis;
- advanced problems of continuous functions, convex and concave functions, differentiable functions and apply them to problems of economic analysis;
- advanced problems of convergence of sequences and series of functions and apply them to problems of economic analysis;
- the basic problem of the measure and integration of functions, understanding the main characteristics and differences between Rieman and Lebesgue integral, and apply them to problems of economic analysis;
- the basic problem of the convergence of sequences of integrals, and apply them to problems of economic analysis;
- the classical theorems of fixed points and apply them to problems of economic analysis;
- advanced problems of correspondences and parametric optimization optimization, and apply them to problems of economic analysis.

# Learning Results

- 1. Mastery of the analysis of functions of one or more variables and in metric or normed spaces, as well as basic concepts of topology in these spaces, in particular by adopting an open finding-solutions-and-counterexamples approach.
- 2. To familiarize students with the mathematical language and the rigor of its statements.
- 3. Mastery of abstract analysis.
- 4. To develop the ability to make assumptions that simplify the problems, by giving partial solutions that may be sufficient for a general problem.
- 5. Mastery of basic mathematical applications in economics, in particular optimization theory, topology, the theorems of continuous functions and correspondences, and fixed point theorems.

# **DESCRIPTION OF CONTENTS: PROGRAMME**

The course is intended to cover most of the mathematical tools required to follow standard first year graduate courses in microeconomics, macroeconomics and statistics. The topics covered are the fundamentals of real analysis and Euclidean spaces, including open and closed sets, compact sets, sequences, series, limits, continuity, differentiability, integration, sequences of functions, and metric and normed spaces. The course also includes fixed point theory for functions and correspondences and the Theorem of the Maximum of Berge.

- 1. Set Theory and the Real Line
  - Ordered Sets. Finite, Countable and Uncountable Sets. The Real Field.
  - Euclidean Spaces. Open, Closed and Compact Sets.
- 2. Numerical Sequences and Series
  - Convergent Sequences. Subsequences. Cauchy Sequences.
  - Convergent and Divergent Series. Absolute Convergence
  - Series of Nonnegative Terms. The Root and Ratio Test.
  - Power Series.
- 3. Continuity
  - Limits of Functions. Continuous Functions.
  - Theorems on Continuous Functions.
  - Monotone Functions. Convex and Concave Functions.
- 4. Differentiation
  - The Derivative of a Real Function.
  - Partial and Directional Derivatives. Differentiability.
  - Inverse and Implicit Function Theorems.
  - Higher Order Derivatives. Taylor Theorem
- 5. Integration
  - Definition and Properties of the Riemann Integral.
  - Fundamental Theorem of Integral Calculus and Barrow; s Rule.
  - Improper Integrals.
  - Introduction to the Lebesque Integral.
- 6. Sequences and Series of Functions
  - Punctual and Uniform Convergence. Equicontinuity.
  - Uniform Convergence and Continuity, Differentiation and Integration.
- 7. Metric Spaces
  - Distance. Metric and Normed Spaces.
  - Open, Closed and Compact Sets.
  - Complete Metric Spaces.
  - Function Spaces.
- 8. Fixed Point Theorems of Functions
  - Theorems of Brower and of Schauder-Tychonoff.
  - Theorem of Banach.
  - Theorem of Tarski
- 9. Correspondences
  - Definition and Properties of Correspondences.
  - Lower and Upper Hemi-Continuous Correspondences.
  - Theorem of the Fixed Point of Kakutani.
- 10. Parametric Optimization
  - Maximum Theorem.
  - Supermodularity and Monotonicity.

## ASSESSMENT SYSTEM

Final Exam 60% Midterm 20% Homework 20%

Extraordinary Exam: best grade of (1) 60% Extraordinary Final Exam + 20% Midterm + 20% Homework and (b) 100% Extraordinary Final Exam.

% end-of-term-examination: 60 % of continuous assessment (assignments, laboratory, practicals...): 40

## **BASIC BIBLIOGRAPHY**

- A de la Fuente Mathematical Methods and Models for Economists, Cambridge University Press, 2005
- AN Kolmogorov y SV Fomin Elements of the Theory of Funcitons and Functional Analysis, Dover, 1990
- C Bergé Espaces Topologiques. Fonctions Multivoques, Dunod, 1966
- FA Ok Analysis with Economic Applications, Princeton University Press, 2007
- H Royden, P. Fitzpatrick Real Analysis. Fourth edition., Pearson, 2010
- K Sydsaeter, P Hammond, A Seierstad, A Strom Further Mathematics for Economic Analysis. Second edition, Prentice Hall, 2008
- NL Stokey, RE Lucas with EC Prescott Recursive Methods in Economic Dynamics, Harvard University Press, 1989
- RK Sundaram First Course in Optimization Theory, Cambridge University Press, 2005
- TM Apostol Mathematical Analysis. Second edition, Addison-Wesley, 1974
- W Rudin Principles of Matematical Analysis. Third edition, McGraw-Hill, 1987
- W Rudin Real and Complex Analysis. Thirs edition, McGraw-Hill, 1987

# ADDITIONAL BIBLIOGRAPHY

- C Aliprantis, O Burkinshaw Problems in Real Analysis. Second edition, Academic Press, 1999
- KR Stromberg An Introduction to Classical Real Analysis, Wadsworth International, 1981
- TM Apostol Calculus I, John Wiley and Sons, 1967
- TM Apostol Calculus II, John Wiley and Sons, 1969
- WH Fleming Functions of Several Variables, Addison Wesley, 1965