uc3m Universidad Carlos III de Madrid

Foundations of Mathematics

Academic Year: (2019 / 2020) Review date: 05-05-2020

Department assigned to the subject:

Coordinating teacher: MARHUENDA HURTADO, FRANCISCO

Type: Compulsory ECTS Credits: 6.0

Year: 1 Semester: 2

OBJECTIVES

The goal of this course is that the student learns to use the mathematical tools needed to succeed in the quantitative subjects (i.e. those in which mathematics plays relevant role to express and organize its contents) he will be taken when he begins his undergraduate studies.

The course will promote a positive attitude towards Mathematics both as a subsidiary content for other sciences (as a tool for modeling different phenomena) and as a science itself organized around logical deductive knowledge where precision is important but also imagination is needed to find answers to problems.

Mainly by solving problems and exercises the student will master the skills and will, as well, grasp the meaning of concepts, integrating both in a meaningful way.

The student after completing this course will be able to define mathematically regions of the real line and the Euclidean plane, work with algebraic expressions and solve all kinds of elementary equations, he will also become familiar with the most important elementary functions, will understand the concept of function and will fluently work with them. He will, as well, identify and use the concepts of the course in practical applications to social sciences.

DESCRIPTION OF CONTENTS: PROGRAMME

The Real Numbers. The Real Line and Order. Properties and Operations.

- Notation for the logic structure of Mathematics : Quantifiers, Implications and Equivalence Relation.
- The real numbers. The real line.
- The absolute value and distance on the real line.
- Intervals: segments and rays. Intersection and union of sets. Inequalities.

The Cartesian Plane.

- Points, distance and the midpoint formula.
- Equations. Straight lines, Circles and Intersection.
- Inequalities and system of inequalities with two unknowns. Solution regions.

Operations with Real Numbers.

- Exponents and radicals. Operations with exponents. Rationalization.
- Exponentials and logarithms. Properties.
- Decimal expression of the real numbers. Approximation. Bounding errors.

Polynomials and Rational expressions.

- Operations with polynomials. Special binomial products.
- Synthetic division.
- Factoring polynomials.
- Rational expressions. Operations.

Equations and System of Equations.

- Polynomial equations. Quadratic and biquadratic equations.
- Rational equations.
- Radical equations.
- Exponential and logarithm equations.
- Linear system of equations with two unknowns. Geometric meaning.
- Gauss elimination method for linear systems.

Functions, properties and basic functions.

- Definition of a function.
- Domain and range of a function. The graph of a function.
- Inverse function.
- Composite function.
- Linear functions.
- Quadratic functions.
- Rational functions.
- Radical functions.
- Piecewise defined functions.
- Function transformation. Translations, dilations and symmetry.
- The absolute value of a function.

Exponential, logarithmic and trigonometric functions.

- Exponential functions.
- Logarithm functions.
- Trigonometrically functions.

Limits of Functions. Continuity.

- Continuity. Types of discontinuities.
- Limits of a function at a point. Continuity.
- Finding limits of a function at a point.
- Infinite limits.
- Asymptotes. Vertical, horizontal and slant (oblique) asymptotes.
- Rational, exponential and logarithmic asymptotes.

Differentiation

- The derivative of a function. Tangent lines. Instantaneous rate of change.
- The derivative function.
- Rules for differentiation.
- The chain rule.
- Application of the derivative. Extreme points of a function. Optimization problems.
- Curve sketching: polynomial and rational functions.

Integration

- Antiderivatives. Finding antiderivatives. Basic integration rules.
- Finding antiderivatives of composite functions.

BASIC BIBLIOGRAPHY

- Knut Sydsæter and Peter J. Hammond, Mathematics for Economic Analysis,, Pearson, 1995