Foundations of Algebra

Academic Year: (2018/2019)

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Department assigned to the subject:

Coordinating teacher: TERAN VERGARA, FERNANDO DE

Type: Compulsory ECTS Credits : 6.0

Year : 1 Semester : 1

REQUIREMENTS (SUBJECTS THAT ARE ASSUMED TO BE KNOWN)

Basic mathematics, including:

- Solution of linear systems of equations with 2 and 3 unknowns.
- Roots of polynomials. Formula for the roots of quadratic polynomials.
- The graph of a linear function.

OBJECTIVES

The student will acquire the following skills:

- 1- Basic skills on mathematical reasoning including:
- Distinguish between implications and equivalence.

- Get familiar with the basic methods of mathematical proving like the method by contradiction or the method of mathematical induction.

- Learn to prove set inclusions and set identities.
- 2- Recognize linear and affine functions and plot them.
- 3- Handle and simplify polynomial equations and know the basic methods of polynomial root-finding.
- 4- To know the meaning of the basic trigonometric ratios.
- 5- To know the ratios of the remarkable angles (0°, 30°, 45°, 60° y 90°).
- 6- Relate the trigonometric ratios of an arbitrary angle with the ones of an angle between 0 and pi radians.
- 7- Relate the trigonometric ratios of complementary and supplementary angles.
- 8- Solve triangules.
- 9- Determine whether a given trigonometric identity is true or not.
- 10- Plot elementary trigonometric functions.
- 11- Identify and plot complex numbers.
- 12- Operate with complex numbers.
- 13- Obtain all different representations of a complex number (binary form, polar form, exponential form).
- 14- Obtain all n nth-roots of a complex number and plot them.
- 15- To know the Fundamental theorem of Algebra.
- 16- Compute the Row Reduced Echelon form of a matrix.
- 17- Solve linear systems using Gaussian elimination.
- 18- Obtain the matrix representation of a linear system.
- 19- Perform arithmetic operations with matrices.
- 20- Obtain the vector expression of a linear system.
- 21- Relate elementary row operations on a matrix with left products by elementary matrices.
- 22- Determine whether a given matrix of low size is invertible or not. In the affirmative case, compute its inverse using
- the algorithm related to the row reduced echelon form of the matrix.
- 23- Apply the recursive definition of the determinant for low-dimensional matrices.
- 24- Relate the determinant of a product of matrices with the determinants of each of the factors.
- 25- To know how the determinant changes when applying elementary row and column operations to the matrix.
- 26- Obtain the determinant of a matrix through an echelon form of the matrix.
- 27- Operate with vectors in R^n.
- 28- Relate the linear independence of a set of vectors with the solution of linear systems.

- 29- Determine whether a given small set of vectors is linearly independent or not.
- 30- To know the notion of spanning set and subspace spanned by a set of vectors.
- 31- Become familiar with the notion of basis of a subspace spanned by a set of vectors.
- 32- Become familiar with the notion of column space of a matrix.
- 33- Become familiar with the notion of null-space of a matrix and relate it with the solution of linear systems.

DESCRIPTION OF CONTENTS: PROGRAMME

WEEK 1: Basic Elements Mathematical notation Basic methods of mathematical proof (Proof by contradiction, the induction method; equation writing) Linear and affine functions Polynomials and polynomial equations

WEEK 2: Trigonometry Trigonometric ratios in the unit circle Relationship between complementary and supplementary angles Relationships between trigonometric ratios. Trigonometric identities Trigonometric functions

WEEKS 3-4: Complex numbers Definition. Binomial form Geometrical representation in R2 Polar and exponential representations. Relation between representations (binomial, polar and exponential) Operations with complex numbers Powers of complex numbers Roots of complex numbers. Geometrical representation The fundamental theorem of algebra

WEEKS 5-6: Linear systems Solving linear systems with 2 unknowns. Geometrical representation. Solving linear systems with 3 unknowns. Geometrical representation. Solving linear systems with n unknowns. Geometrical representation. Gaussian elimination Basic definitions Echelon and reduced echelon forms homogeneous systems Existence and uniqueness theorem Solutions in parametric form

WEEKS 7-8: Matrices Basic definitions Matrix operations Matrix representation of a linear system Vector representation of a linear system Elementary matrices and elementary row operations. Relation to linear systems Inverse of a matrix

WEEK 9: Determinants Definition Determinant of a triangular matrix Determinant of the product of square matrices Determinant of the transpose of a matrix Elementary row operations in a determinant Determinant of a matrix using the echelon form of a matrix

WEEK 10: Vectors in Rn. Basic operations Linear independence Spanning sets. Span of a set of vectors Bases Column space of a matrix Null space of a matrix Revisiting linear systems

ASSESSMENT SYSTEM

The final grade will come from:

- Mid-term assessments (50%).
- Final exam (50%).

The mid-term assessment will consist of several mid-term test. Some homework could also be included in this part of the assessment.

BASIC BIBLIOGRAPHY

- David Poole Linear Algebra: A modern introduction, Cengage Brooks/Cole, 2011